Last Name: SOLUTIONS

First Name: _____

Student ID: _____

Assignment 1

Please answer all of the following questions in the space provided. Write clearly and make sure to use correct notation. Please state any variables, distributions and assumptions that you are using/making.

Question 1. *(5 marks)* Hexadecimal is a numeral system with base 16 (instead of base 10). The digits used are 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, A, B, C, D, E, F (With A=10, B=11, C=12, D=13, E=14, F=15). Suppose we propose an experiment where a digit in hexadecimal is selected at random. We define the following events:

$A = \{0,1,2,3,4,5,6,7\}$

$B = \{5,6,7,8,9,A,B,F\}$

$C = \{1,5,B,F\}$

Are A, B, and C mutually independent events? Make a venn diagram with probabilities to describe this situation.

$P(A) = \frac{8}{16} = \frac{1}{2}$ $\quad P(B) = \frac{8}{16} = \frac{1}{2}$ $\quad P(C) = \frac{4}{16} = \frac{1}{4}$

$A \cap B = \{5,6,7\}$ $\quad A \cap C = \{1,5\}$, $\quad B \cap C = \{5,B\}$

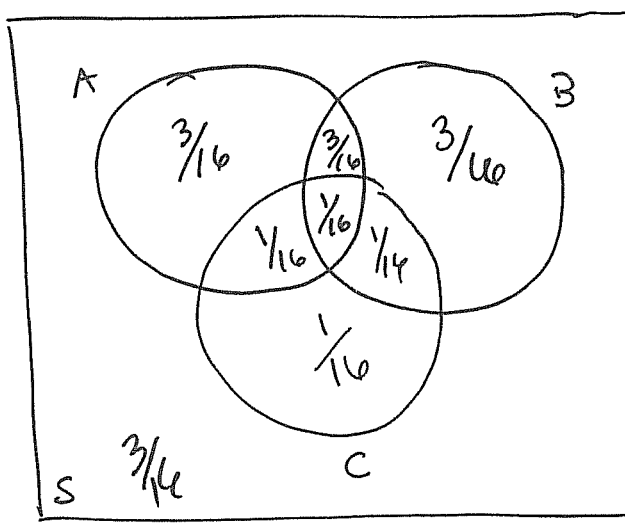
$A \cap B \cap C = \{5\}$

$P(A \cap B) = \frac{4}{16} = \frac{1}{4} = P(A) \cdot P(B)$

$P(A \cap C) = \frac{2}{16} = \frac{1}{8} = P(A) \cdot P(C)$

$P(B \cap C) = \frac{2}{16} = \frac{1}{8} = P(B) \cdot P(C)$

$P(A \cap B \cap C) = \frac{1}{16} = P(A)P(B)P(C)$



YES, A, B AND C ARE MUTUALLY INDEPENDENT.

Question 2. A shopkeeper is going to leave her store for 5 minutes. The number of customers entering her store in a given hour follows a Poisson distribution with mean 6.

(a) *(3 marks)* What is the probability that exactly 10 people show up between 9:00am and 11:00am.

LET x BE THE NUMBER OF PEOPLE THAT COME INTO THE STORE BETWEEN 9:00AM AND 11:00AM (2 HOURS)

$\therefore \mu = 12$ PEOPLE

$$P(10) = \frac{\mu^{10} e^{-\mu}}{10!} = \frac{12^{10} e^{-12}}{10!} = 0.105 \quad \text{SO } 10.5\%$$

(b) *(5 marks)* If too few customers come into the store during the first hour of opening, the shopkeeper sends a clerk home for the day. She sends a clerk home 28% of the time. What is the number of customers such that if less than that number come into the shop in the first hour, she sends a clerk home?

LET x = # OF CUSTOMERS IN FIRST HOUR, $\mu = 6$ $\therefore P(x) = \frac{6^x e^{-6}}{x!}$

x	$P(x)$
0	0.002
1	0.015
2	0.045
3	0.089
4	0.134

$\therefore P(0) + P(1) + P(2) + P(3) + P(4) = 0.002 + 0.015 + 0.045 + 0.089 + 0.134$

$= 0.283 \approx 28\%$

SHE SHOULD SEND THE CERK HOME IF 4 OR LESS PEOPLE SHOW UP (LESS THAN 5).

Question 3. *(5 marks)* An airline has the policy of overbooking their flights. A flight from Montreal to Toronto has 88 seats. The airline has booked 93 tickets on this flight but the airline has learned that 6 % of the time passengers don't show up for their flights.

(a) *(3 marks)* How many people do you expect to show up for the flight?

BINOMIAL DISTRIBUTION, $n = 93$ LET X = # OF PEOPLE WHO SHOW UP (OUT OF 93)

$p = 0.94$ $q = 0.06$

EXPECTED VALUE $\mu = 93(0.94) = 87.4$

(b) *(4 marks)* What are the chances that there will be a seat for everyone that shows up?

$P(x \leq 88) = 1 - P(x \geq 89) = 1 - P(89) - P(90) - P(91) - P(92) - P(93)$

$P(89) = {}_{93}C_{89}(0.94)^{89}(0.06)^4 = {}_{93}C_4(0.94)^{89}(0.06)^4 = 0.154$

$P(90) = {}_{93}C_{90}(0.94)^{90}(0.06)^3 = {}_{93}C_3(0.94)^{90}(0.06)^3 = 0.107$

$P(91) = {}_{93}C_{91}(0.94)^{91}(0.06)^2 = {}_{93}C_2(0.94)^{91}(0.06)^2 = 0.055$

$P(92) = {}_{93}C_{92}(0.94)^{92}(0.06)^1 = {}_{93}C_1(0.94)^{92}(0.06)^1 = 0.018$

$P(93) = {}_{93}C_{93}(0.94)^{93}(0.06)^0 = {}_{93}C_0(0.94)^{93}(0.06)^0 = 0.003$

0.337

$\therefore P(x \leq 88) = 1 - 0.337 = 0.663$

A 66.3% CHANCE THERE WILL BE A SEAT FOR EVERYONE THAT SHOWS UP.

Question 4. A bag with 200 jellybeans contains 48 red jellybeans. Suppose a handful of seven jellybeans are pulled from the bag.

(a) *(4 marks)* If we were to ask what the chances are that we have three red jellybeans in our hand, what type of probability distribution would we use to solve this problem? What are the chances that three out of the seven are red?

HYPERGEOMETRIC DISTRIBUTION: $N = 200$, $M = 48$, $X = \#$ OF RED JELLYBEANS FROM SAMPLE

$n = 7$

$$\therefore P(3) = \frac{{}_{48}C_3 \cdot {}_{152}C_4}{{}_{200}C_7} = 0.162 \quad \text{OR} \quad 16.2\%$$

(b) *(4 marks)* Is it possible to use another type of distribution to approximate this probability? Why or why not? If it is, do so.

YES, SINCE $\frac{n}{N} = \frac{7}{200} = 0.035 \leq 0.05$

(SAMPLE IS LESS THAN 5% OF POPULATION)

$\therefore$ WE CAN USE BINOMIAL DISTRIBUTION APPROXIMATION.

$n = 7$, $x = 3$, $p = \frac{48}{200} = 0.24$ $\quad q = 1 - 0.24 = 0.76$

$$\therefore P(3) = {}_7C_3 (0.24)^3 (0.76)^4$$

$$= 0.161$$

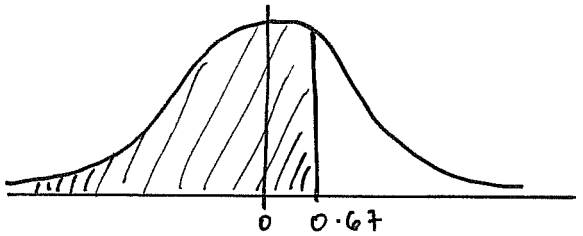
APPROXIMATELY 16.1%

Question 5. *(4 marks)* Use the z-table to find the following proportions. Remember to always draw the standard normal curve and shade the area that you're finding.

(a) $P(z < 0.67)$ $= 0.5 + P(0 < z < 0.67)$

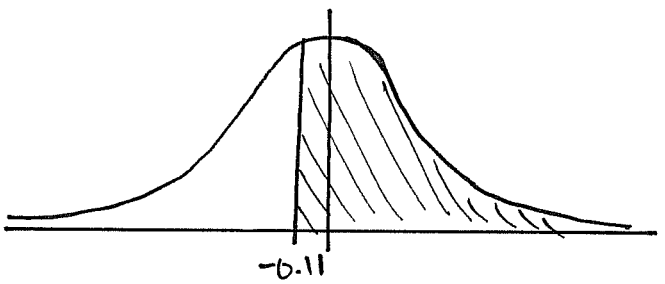
$= 0.5 + 0.2486$

$= 0.7486$

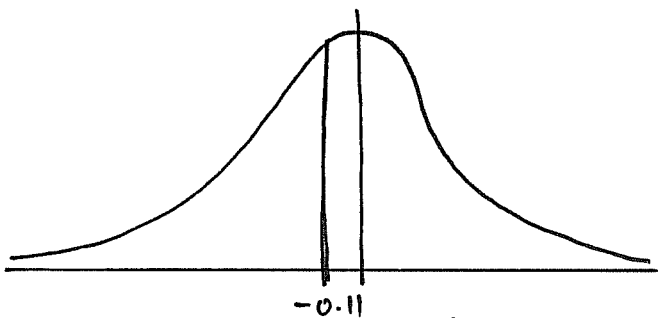


(b) $P(z > -0.11) =$ $0.5 + P(0 < z < 0.1)$

$= 0.5 + 0.0438 = 0.5438$



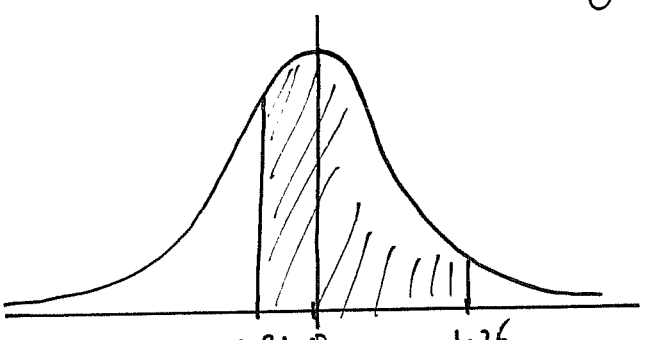
(c) $P(z = -0.11) = 0$



(d) $P(-0.31 < z < 1.25)$ $= P(0 < z < 0.31) + P(0 < z < 1.25)$

$= 0.1217 + 0.3944$

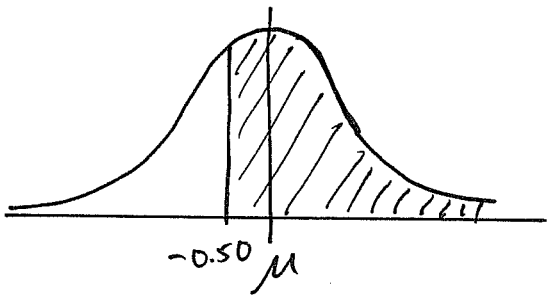
$= 0.5161$



Question 6. *(5 marks)* The heights of Canadian men are approximately normally distributed, with a mean of 68.5in and a standard deviation of around 3in.

(a) What proportion of Canadian men are taller than 5'7"?

$5'7" = 67$ in $\Rightarrow$ $z = \dfrac{67 - 68.5}{3} = -0.50$



$P(x > 67) = P(z > -0.50)$

$= 0.5 + P(0 < z < 0.50)$

$= 0.5 + 0.1915$

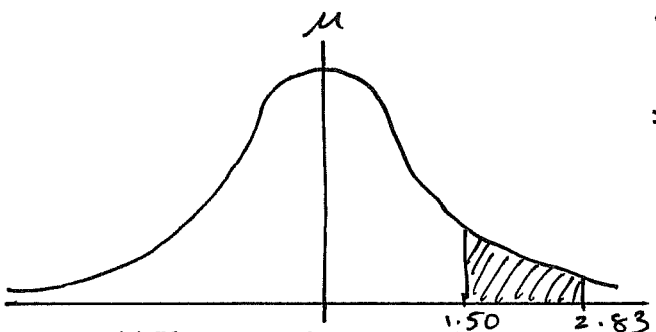
$= 0.6915$

69.15% OF CANADIAN MEN.

(b) What proportion of Canadian men are between 6'1" and 6'5"?

$6'1" = 73$in $\Rightarrow$ $z = \dfrac{73 - 68.5}{3} = 1.50$

$6'5" = 77$in $\Rightarrow$ $z = \dfrac{77 - 68.5}{3} = 2.83$



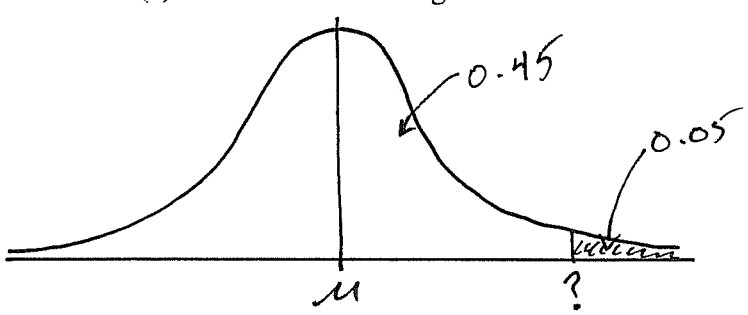
$P(73 < x < 77) = P(1.50 < z < 2.83)$

$= P(0 < z < 2.83) - P(0 < z < 1.50)$

$= 0.4977 - 0.4332$

$= 0.0645$ $\Rightarrow$ 6.45% OF CANADIAN MEN

(c) If someone is among tallest 5% of all Canadian men, he must be at least what height?



$0.4505 = P(0 < z < 1.65)$ FROM CHART

$1.65 = \dfrac{x - 68.5}{3}$

$\Rightarrow$ $x = 73.45$

HE MUST BE AT LEAST 73.45in TALL