Last Name: SOLUTIONS.

First Name: ____________

Student ID: ____________

Test 2

The time allowed for this test is 1 hour and 45 minutes. Please answer all questions in the space provided. A formula sheet is available upon request but if you decide not to use it you will be awarded 2 bonus marks. Remember to write clearly and use correct notation. If you use it, remember to always draw the standard normal curve and shade the area that you're finding

Question 1. *(4 marks)* Is the function

$$P(x) = \begin{cases} \frac{8}{15} & \text{if} \quad x = 1 \\ \frac{1}{2} \cdot P(x-1) & \text{if} \quad x > 1 \end{cases}$$

for $x = 1, 2, 3, 4$, a probability function?

x	$P(x)$
1	$8/15$
2	$1/2 \cdot \frac{8}{15} = \frac{4}{15}$
3	$\frac{1}{2} \cdot \frac{4}{15} = \frac{2}{15}$
4	$\frac{1}{2} \cdot \frac{2}{15} = \frac{1}{15}$

$\therefore 0 \leq P(x) \leq 1$ FOR $x = 1, 2, 3, 4$

AND $P(1) + P(2) + P(3) + P(4) = \frac{8}{15} + \frac{4}{15} + \frac{2}{15} + \frac{1}{15} = 1$

$\therefore$ YES $P(x)$ IS A PROBABILITY FUNCTION.

Question 2. *(5 marks)* An audio amplifier contains six transistors. It has been ascertained that two of the transistors are faulty but it is not known which two. Amy removes three transistors at random, and inspects them. What is the probability that amy has removed the two faulty transistors? (State what the random variable is, which distribution you are using and why.)

HYPERGEOMETRIC DISTRIBUTION: SAMPLING WITHOUT REPLACEMENT, NOT INDEPENDENT

LET $X =$ # OF FAULTY TRANSISTORS OUT OF $n=3$

$$\therefore P(2) = \frac{\binom{2}{2}\cdot\binom{4}{1}}{\binom{6}{3}}$$

$$= \frac{(1)(4)}{20} = \frac{1}{5} = 0.20.$$

THE PROBABILITY THAT AMY HAS REMOVED THE TWO FAULTY TRANSISTORS IS 20%

Question 3. *(5 marks)* A light bulb company wants to make a guarantee to it's customers that if they buy a pack of 8 lightbulbs and more than a n stop working within the first year, they will replace the pack. The company knows that 10% of the lightbulbs they are producing stop working within the first year. If they only want to replace approximately 4% of the packs of lightbulbs, what should the number n be? (State what the random variable is, which distribution you are using and why.)

EACH SAMPLE IS INDEPENDENT $\Rightarrow$ BINOMIAL DISTRIBUTION

$p = 0.10$, $q = 0.90$ LET $X =$ # OF DEFECTIVE BULBS OUT OF 6.

$P(0) = {}_8C_0 (0.10)^0 (0.90)^8 = 0.43046721$

$P(1) = {}_8C_1 (0.10)^1 (0.90)^7 = 0.38263752$

$P(2) = {}_8C_2 (0.10)^2 (0.90)^6 = 0.14880348$

SUM 0.961

$\therefore P(x > 2) = 1 - 0.961 = 0.0039 \approx 4\%$

$\therefore n = 2$, IF MORE THAN 2 STOP WORKING WITHIN THE FIRST YEAR WILL REPLACE THE PACK.

Question 4. *(6 marks)* Based upon past experience, 60% of all customers at the Dawson book store pay for their purchases with a debit card. If a random sample of 200 customers is selected, what is the probability that at least 110 of the customers used their debit card. (You may use an approximation if possible. If you do use an approximation make sure to justify its use.)

THIS A BINOMIAL DISTRIBUTION (INDEPENDENT SAMPLES)
BUT WE CAN APPROXIMATE WITH A NORMAL DISTRIBUTION

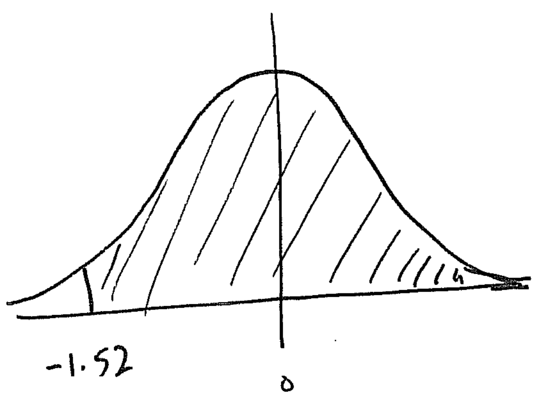
SINCE $n \cdot p = 200(0.60) = 120 \geq 5$
$n \cdot (1-p) = 200(0.40) = 80 \geq 5$

CONTINUITY CORRECTION $x \geq 109.5$

EXPECTED VALUE $= 120$

$\sigma = \sqrt{n \cdot p \cdot q} = \sqrt{200(0.60)(0.40)} = \sqrt{48} = 6.9282$

$$z = \frac{109.5 - 120}{6.9282} = -1.52$$



$P(z \geq -1.52)$
$= P(0 \leq z \leq 1.52) + 0.5$
$= 0.4357 + 0.5$
$= 0.9357$

APPROXIMATELY 93.57%

Question 5. **(a)** *(2 marks)* The central limit theorem say three main facts about sample distributions of sample means. State any two of these facts.

1) FOR SAMPLING DISTRIBUTIONS OF SAMPLE MEANS, $\mu_{\bar{x}} = \mu_x$

2) $\sigma_{\bar{x}}$ DECREASES AS n INCREASES

3) DISTRIBUTIONS BECOME MORE NORMAL AS n INCREASES

(b) *(2 marks)* Give an example of an unbiased statistic and state why it is an unbiased statistic.

MEAN IS AN UNBIASED STATISTIC SINCE $\mu_{\bar{x}} = \mu_x$.

Question 6. *(2 marks)* For a hypothesis test, explain what the probabilities α and β are.

$\alpha = P(\text{REJECT } H_0 \mid H_0 \text{ IS TRUE})$ ← PROBABILITY OF TYPE I ERROR

$\beta = P(\text{ACCEPT } H_0 \mid H_0 \text{ IS FALSE})$ ← PROBABILITY OF TYPE II ERROR.

Question 7. (a) *(5 marks)* The distribution of blood cholesterol level in the population of Canadian men aged 18 to 34 is close to normal, with a standard deviation of 0.9 mmol/L. You measure the blood cholesterol level of 14 Canadian men in this age bracket who are cross-country runners, and find that the mean cholesterol level is 4.4 mmol/L. Construct a 92% confidence interval for the mean blood cholesterol level of all Canadian male cross-country runners in this age range.

APPROXIMATELY NORMAL $\Rightarrow$ WE CAN USE Z-TABLE.

$\alpha = 0.08 \Rightarrow \alpha/2 = 0.04 \Rightarrow z(\alpha/2) = 1.755$

$E = z(\alpha/2) \cdot \frac{\sigma}{\sqrt{n}} = 1.755\left(\frac{0.9}{\sqrt{14}}\right) = 0.422$

$\therefore$ 92% C.I.E :

$3.978 < \mu < 4.822$ mmol/L

(b) *(4 marks)* Suppose you wanted to construct a 92% confidence interval for the mean blood cholesterol level of all Canadian male cross-country runners in this age range but you wanted the error to be at most 0.3 mmol/L. How would you do this?

INCREASING SAMPLE SIZE

$$n = \left[\frac{z(\alpha/2)\cdot\sigma_x}{E}\right]^2 = \left[\frac{(1.755)(0.9)}{0.3}\right]^2$$

$$= 27.72$$

$\therefore$ USING A SAMPLE SIZE 28 WILL GIVE US A 92% CONFIDENCE INTERVAL WITH ERROR AT MOST 0.3 mmol/L.

Question 8. *(5 marks)* A random sample of size 12 of FICO credit ratings had mean 761. The current mean FICO credit rating was reported in a newspaper as 715. FICO credit ratings are known to be approximately normally distributed with a standard deviation of 58.3. Can we conclude that these FICO credit ratings from the sample did not come from a population with a mean of 715, at the 5% level of signifcance? Use the probability value approach to hypothesis testing.)

APPROXIMATELY NORMALLY DISTRIBUTED $\Rightarrow$ WE CAN USE Z-TABLE.

$$\bar{x} = 761 \Rightarrow \quad z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{761 - 715}{58.3/\sqrt{12}} = 2.73$$

TWO TAIL TEST:

$$P(z < -2.73) + P(z > 2.73)$$

$$= 1 - 2P(0 \le z \le 2.73)$$

$$= 1 - 2(0.4968)$$

$$= 0.0064 \quad < 0.05 = \alpha$$

REJECT H_0, AT 5% SIGNIFICANCE THESE FICO RATINGS DID NOT COME FROM A POPULATION WITH MEAN 715.

Question 9. **(a)** *(5 marks)* Tests of older baseballs show that when dropped 24 ft onto a concrete surface, they bounced an average of 235.8 cm. In a test of 40 new baseballs, the bounce heights had a mean of 235.4 cm. Assume the standard deviation for bounce heights is known to be 4.5 cm. Is this sample data sufficient evidence to conclude that the newer baseballs are less bouncy than older ones, at the 5% significance level? (Use the classical approach to hypothesis testing.)

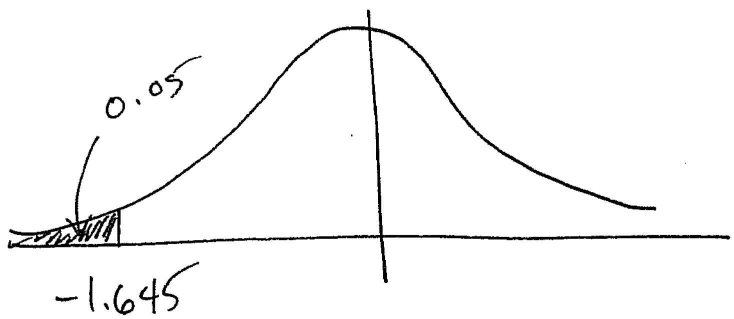
$n = 40 \geq 30$ $\therefore$ WE CAN USE Z-TABLE

$H_0: \mu \geq 235.8$

$H_a: \mu < 235.8$

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{235.4 - 235.8}{4.5/\sqrt{40}} = -0.56$$

$\alpha = 0.05 \Rightarrow z(\alpha) = -1.645$



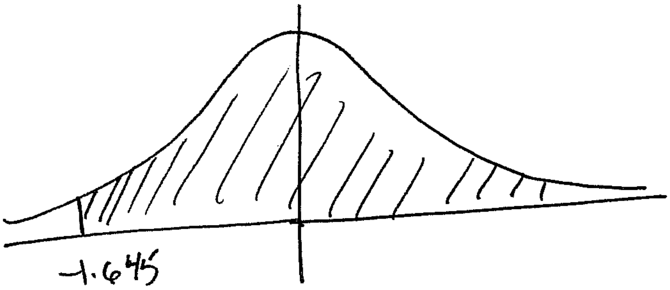
REJECTION REGION: $z < -1.645$

$\therefore$ $z = -0.56$ NOT IN REJECTION REGION.

WE CANNOT CONCLUDE THAT THE NEWER BALLS ARE ANY LESS BOUNCY (AT 5% SIGNIFICANCE)

(b) *(5 marks)* Suppose for the population new baseballs, the bounce heights actually had a mean of 233.4 cm. For the test constructed in part (a), what is β in this case?

ACCEPTANCE REGION WAS $Z > -1.645$

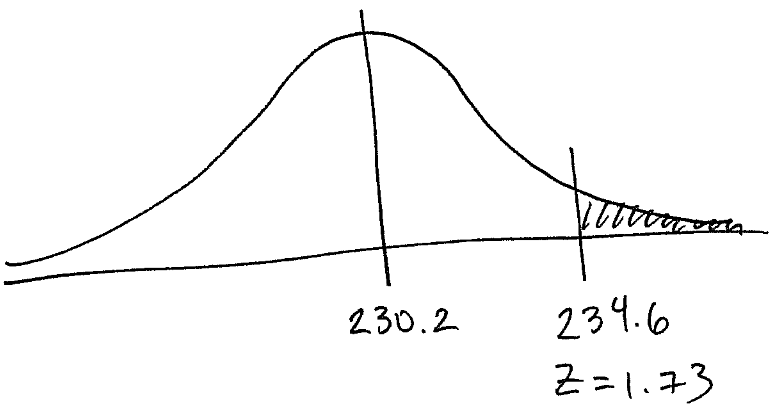


$$235.8 - 1.645 \cdot \frac{(4.5)}{\sqrt{40}} = 234.6$$

$\therefore \bar{x} > 234.6$

NEW Z VALUE.

$$Z = \frac{234.6 - 233.4}{4.5/\sqrt{40}} = 1.73$$



$$\beta = P(Z > 1.73) = 0.5 - P(0 < Z < 1.73)$$

$$= 0.5 - 0.4582$$

$$= 0.0418$$