Dawson College: **Probability and Statistics**: 201-BZS-05

April 29, 2013

Last Name: SOLUTIONS

First Name: ____________

Student ID: ____________

Test 3

The time allowed for this test is 1 hour and 45 minutes. Please answer all questions in the space provided. A formula sheet is available upon request but if you decide not to use it you will be awarded 3 bonus marks. Remember to write clearly and use correct notation. If you use a distribution make sure to draw it and shade the relevant area.

Question 1. *(6 marks)* Ten randomly selected shut-ins were each asked to list how many hours of television they watched per week. The results are

82 66 90 84 75 88 80 94 110 91

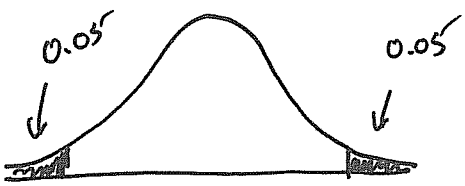
Determine the 90% confidence interval estimate for the mean number of hours of television watched per week by shut-ins. Assume the number of hours is normally distributed.

$\bar{x} = 86 \quad s = 11.84$

NORMALLY DISTRIBUTED $\Rightarrow$ USE t-TABLE

$df = 10 - 1 = 9$

$t(0.05) = 1.83$



$$E = 1.83 \frac{(11.84)}{\sqrt{10}} = 6.85$$

$\therefore$ 95% C.I.E. $79.15 < \mu < 92.85$ hrs

Question 2. *(4 marks)* A bank wants to know what proportion of its checking-account customers have used at least one other service provided by the bank within the last six months. How large a sample will be needed to estimate the true proportion to within 5% at the 98% level of confidence?

$\alpha/2 = 0.01$, $E = 5\% = 0.05$

$z(\alpha/2) = 2.33$

$\therefore \quad n = [z(\alpha/2)]^2 \cdot \dfrac{p(1-p)}{E^2}$

$= (2.33)^2 \dfrac{(0.5)(0.5)}{(0.05)^2}$

$= 542.892$

$\therefore$ SAMPLE SIZE SHOULD BE 543

Question 3. *(6 marks)* An experiment was conducted to compare the mean absorptions of two drugs in specimens of muscle tissue. Seventy-two tissue specimens were randomly divided into two equal groups. Each group was tested with one of the two drugs. The sample results were $\bar{x}_A = 8.48$ $\bar{x}_B = 8.5$, $s_A = 0.11$, and $s_B = 0.10$. Use the probabilistic approach to hypothesis testing with $\alpha = 0.05$ to determine if there is a difference in the mean absorptions of the two drugs, (ASSUME NORMAL DISTRIBUTIONS)

$H_0: \mu_A - \mu_B = 0$ ∴ TWO TAIL TEST

$H_a: \mu_A - \mu_B \neq 0$

$$df = \frac{\left(\frac{(0.11)^2}{36} + \frac{(0.10)^2}{36}\right)^2}{\frac{(0.11)^4}{36^2(35)} + \frac{(0.10)^4}{36^2(35)}} = 69.89 \rightarrow 69$$

TEST STATISTIC

$$t = \frac{(\bar{x}_A - \bar{x}_B) - 0}{\sqrt{\frac{s_A^2}{n_A} + \frac{s_B^2}{n_B}}} = -0.81$$

$$P(t < -0.81) + P(t > 0.81) = 2P(t > 0.81) = 2(0.214) > \alpha/2 = 0.025$$

∴ FAIL TO REJECT H_0.

AT 5% SIGNIFICANCE, WE CANNOT CONCLUDE THAT THERE IS A DIFFERENCE IN MEAN ABSORPTIONS OF THE TWO DRUGS.

Question 4. *(7 marks)* The quality of latex paint is monitored by measuring different characteristics of the paint. One characteristic of interest is the particle size. Two different types of disc centrifuges (JLDC, Joyce Loebl Disc Centrifuge, and the DPJ, Dwight P. Joyce disc) are used to measure the particle size. It is thought that these two methods yield different measurements. EIGHT readings were taken from the same batch of latex paint using both the JLDC and the DPJ discs

JLDC	4714	4601	4696	4896	4905	4870	4987	5144
DPJ	4295	4271	4326	4530	4618	4779	4752	4744
d	419	330	370	366	287	91	235	400

Assuming particle size is normally distributed, determine whether there is a significant difference between the readings at the 0.01 level of significance.

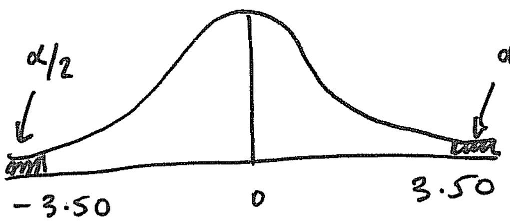
$\bar{d} = 312.25$ $\quad S_d = 107.698$

$df = 8-1 = 7$

$H_0: \mu_d = 0$

$H_a: \mu_d \neq 0$ $\quad$ TWO TAIL TEST

$t(\alpha/2) = t(0.005) = 3.50$



← REJECTION REGION

TEST STATISTIC

$$t = \frac{\bar{d} - \mu_d}{S_d/\sqrt{n}} = \frac{312.25 - 0}{107.698/\sqrt{8}} = 8.20$$ IN REJECTION REGION

REJECT Ho.

∴ AT 0.01 SIGNIFICANCE, WE CAN CONCLUDE THAT THERE IS A DIFFERENCE BETWEEN THE MEANS OF THE READINGS

Question 5. *(6 marks)* Keirin track cycling is a popular betting sport in Japan. Does the starting position of the rider effect the rider's chances of winning? Use the data below with $\alpha = 5\%$ to answer this question.

Position	1	2	3	4	5
Number of wins	28	17	16	26	9

LET p_i BE THE POPULATION PROPORTION OF WINS IN POSITION i.

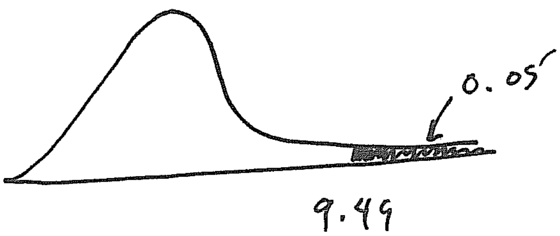
H_0: $p_i = \frac{1}{5}$ FOR $i = 1, \ldots, 5$

H_a: $p_i \neq \frac{1}{5}$ FOR SOME $i \in \{1, \ldots, 5\}$

SO $e_i = \frac{1}{5}(96) = 19.2$ FOR ALL $i = \{1, \ldots, 5\}$ SO $e_i > 5$ WE CAN USE χ^2 DISTRIBUTION

$df = 5 - 4 = 4$

$\chi^2(0.05) = 9.49$ $\therefore$ REJECTION REGION $\chi^2 > 9.49$



TEST STATISTIC

$$\chi^2 = \sum_{i=1}^{5} \frac{(f_i - e_i)^2}{e_i} = \frac{(28-19.2)^2}{19.2} + \frac{(17-19.2)^2}{19.2} + \frac{(16-19.2)^2}{19.2} + \frac{(26-19.2)^2}{19.2}$$

$$+ \frac{(9-19.2)^2}{19.2} = 12.65$$

$\therefore$ IN REJECTION REGION

AT 5% WE CAN CONCLUDE THAT STARTING POSITION HAS AN EFFECT ON THE RIDERS CHANCE OF WINNING.

Question 6. *(6 marks)* The article "Making Up for Lost Time" reported that more than half of the country's workers aged 45 to 64 want to quit work before they reach age 65. Suppose you conduct a survey of 1000 randomly chosen workers in order to test if this figure is too high. 460 of the 1000 sampled want to quit work before age 65. Use $\alpha = 0.01$.

$p' = \frac{460}{1000} = 0.46$

$0.46(1000) = 460 > 5$

$0.64(1000) = 640 > 5$

$\therefore$ WE CAN USE Z-TABLE.

$H_0: p_0 \geq 0.5$

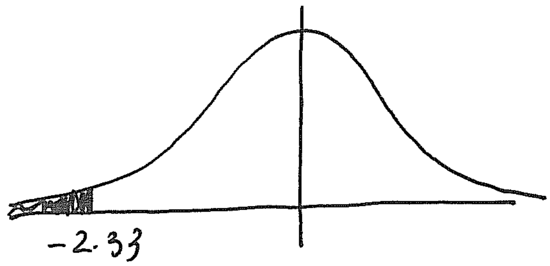
$H_a: p_0 < 5$

LEFT TAIL TEST

$\alpha = 0.01$

$-z(\alpha) = -2.33$

REJECTION REGION



TEST STATISTIC:

$$z = \frac{p' - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} = \frac{0.46 - 0.5}{\sqrt{\frac{0.5(0.5)}{1000}}} = -2.53$$

IN REJECTION REGION

REJECT Ho

$\therefore$ AT 5% SIGNIFICANCE WE CAN CONCLUDE THAT THIS FIGURE IS TOO HIGH.

Question 7. *(7 marks)* A random sample of 500 car batteries revealed the following distribution of battery life (in years).

Life in years	Observed frequencies	Z-RANGE	AREA	e_i
$x < 2$	106	$z < -0.82$	0.2033	103.05
$2 \leq x < 3$	170	$-0.82 \leq z < 0.21$	0.3771	188.55
$3 \leq x < 4$	165	$0.21 \leq z < 1.24$	0.3095	154.75
$4 \leq x$	59	$1.24 \leq z$	0.1075	53.75

For this sample the sample mean is $\bar{x} = 2.80$ and sample standard deviation is $s = 0.97$. At $\alpha = 0.05$, does battery life follow a normal distribution?

SINCE $e_i > 5$ WE CAN USE χ^2

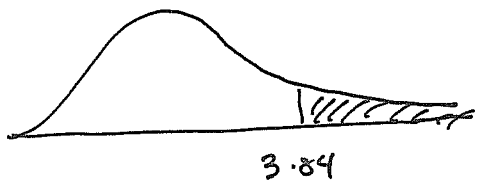
$$df = 4 - 1 - 1 - 1 = 1$$

USING $\bar{x}$ USING S

H_0: SAMPLE COMES FROM NORMAL POPULATION

H_a: SAMPLE DOES NOT COME FROM NORMAL POPULATION

$\chi^2(0.05) = 3.84$ REJECTION REGION



TEST STATISTIC

$$\chi^2 = \frac{(106 - 103.05)^2}{103.05} + \frac{(170 - 188.55)^2}{188.55} + \frac{(165 - 154.75)^2}{154.75}$$

$$+ \frac{(59 - 53.75)^2}{53.75} = 3.10$$ NOT IN REJECTION REGION

AT 5% SIGNIFICANCE, WE CANNOT CONCLUDE THAT THIS DATA DID NOT COME FROM A NORMAL POPULATION.