

$$1. f(x) = 6x^2 - 8x + 3 \Rightarrow F(x) = 6\frac{x^{2+1}}{2+1} - 8\frac{x^{1+1}}{1+1} + 3x + C = 2x^3 - 4x^2 + 3x + C$$

$$\text{Check: } F'(x) = 2 \cdot 3x^2 - 4 \cdot 2x + 3 + 0 = 6x^2 - 8x + 3 = f(x)$$

$$2. f(x) = 1 - x^3 + 12x^5 \Rightarrow F(x) = x - \frac{x^{3+1}}{3+1} + 12\frac{x^{5+1}}{5+1} + C = x - \frac{1}{4}x^4 + 2x^6 + C$$

$$3. f(x) = 5x^{1/4} - 7x^{3/4} \Rightarrow F(x) = 5\frac{x^{1/4+1}}{\frac{1}{4}+1} - 7\frac{x^{3/4+1}}{\frac{3}{4}+1} + C = 5\frac{x^{5/4}}{5/4} - 7\frac{x^{7/4}}{7/4} + C = 4x^{5/4} - 4x^{7/4} + C$$

$$4. f(x) = 2x + 3x^{1.7} \Rightarrow F(x) = x^2 + \frac{3}{2.7}x^{2.7} + C = x^2 + \frac{10}{9}x^{2.7} + C$$

$$5. f(x) = \sqrt[3]{x} + \frac{5}{x^6} = x^{1/3} + 5x^{-6} \text{ has domain } (-\infty, 0) \cup (0, \infty), \text{ so}$$

$$F(x) = \begin{cases} \frac{x^{1/3+1}}{\frac{1}{3}+1} + 5\frac{x^{-6+1}}{-6+1} + C_1 = \frac{3}{4}x^{4/3} - x^{-5} + C_1 & \text{if } x < 0 \\ \frac{3}{4}x^{4/3} - x^{-5} + C_2 & \text{if } x > 0 \end{cases}$$

See Example 1(b) for a similar problem.

$$6. f(x) = \sqrt[4]{x^3} + \sqrt[3]{x^4} = x^{3/4} + x^{4/3} \Rightarrow F(x) = \frac{x^{7/4}}{7/4} + \frac{x^{7/3}}{7/3} + C = \frac{4}{7}x^{7/4} + \frac{3}{7}x^{7/3} + C$$

$$7. f(u) = \frac{u^4 + 3\sqrt{u}}{u^2} = \frac{u^4}{u^2} + \frac{3u^{1/2}}{u^2} = u^2 + 3u^{-3/2} \Rightarrow$$

$$F(u) = \frac{u^3}{3} + 3\frac{u^{-3/2+1}}{-3/2+1} + C = \frac{1}{3}u^3 + 3\frac{u^{-1/2}}{-1/2} + C = \frac{1}{3}u^3 - \frac{6}{\sqrt{u}} + C$$

$$8. g(x) = \frac{5 - 4x^3 + 2x^6}{x^6} = 5x^{-6} - 4x^{-3} + 2 \text{ has domain } (-\infty, 0) \cup (0, \infty), \text{ so}$$

$$G(x) = \begin{cases} 5\frac{x^{-5}}{-5} - 4\frac{x^{-2}}{-2} + 2x + C_1 = -\frac{1}{x^5} + \frac{2}{x^2} + 2x + C_1 & \text{if } x < 0 \\ -\frac{1}{x^5} + \frac{2}{x^2} + 2x + C_2 & \text{if } x > 0 \end{cases}$$

$$9. g(\theta) = \cos \theta - 5 \sin \theta \Rightarrow G(\theta) = \sin \theta - 5(-\cos \theta) + C = \sin \theta + 5 \cos \theta + C$$

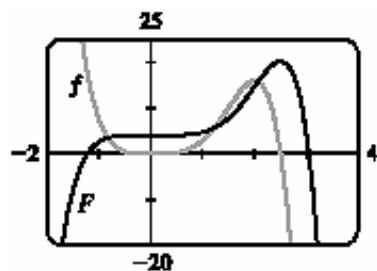
$$11. f(x) = 2x + 5(1-x^2)^{-1/2} = 2x + \frac{5}{\sqrt{1-x^2}} \Rightarrow F(x) = x^2 + 5 \sin^{-1} x + C$$

$$12. f(x) = \frac{x^2 + x + 1}{x} = x + 1 + \frac{1}{x} \Rightarrow F(x) = \begin{cases} \frac{1}{2}x^2 + x + \ln|x| + C_1 & \text{if } x < 0 \\ \frac{1}{2}x^2 + x + \ln|x| + C_2 & \text{if } x > 0 \end{cases}$$

$$13. f(x) = 5x^4 - 2x^5 \Rightarrow F(x) = 5 \cdot \frac{x^5}{5} - 2 \cdot \frac{x^6}{6} + C = x^5 - \frac{1}{3}x^6 + C.$$

$$F(0) = 4 \Rightarrow 0^5 - \frac{1}{3} \cdot 0^6 + C = 4 \Rightarrow C = 4, \text{ so } F(x) = x^5 - \frac{1}{3}x^6 + 4.$$

The graph confirms our answer since $f(x) = 0$ when F has a local maximum, f is positive when F is increasing, and f is negative when F is decreasing.

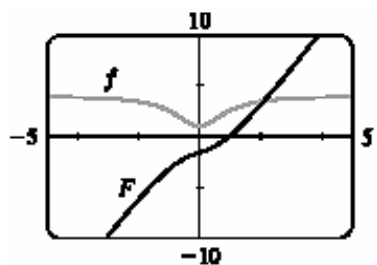


$$14. f(x) = 4 - 3(1+x^2)^{-1} = 4 - \frac{3}{1+x^2} \Rightarrow F(x) = 4x - 3 \tan^{-1} x + C.$$

$$F(1) = 0 \Rightarrow 4 - 3\left(\frac{\pi}{4}\right) + C = 0 \Rightarrow C = \frac{3\pi}{4} - 4, \text{ so}$$

$$F(x) = 4x - 3 \tan^{-1} x + \frac{3\pi}{4} - 4. \text{ Note that } f \text{ is positive and } F \text{ is increasing on } \mathbb{R}.$$

Also, f has smaller values where the slopes of the tangent lines of F are smaller.



$$15. f''(x) = 6x + 12x^2 \Rightarrow f'(x) = 6 \cdot \frac{x^2}{2} + 12 \cdot \frac{x^3}{3} + C = 3x^2 + 4x^3 + C \Rightarrow$$

$$f(x) = 3 \cdot \frac{x^3}{3} + 4 \cdot \frac{x^4}{4} + Cx + D = x^3 + x^4 + Cx + D \quad [C \text{ and } D \text{ are just arbitrary constants}]$$

$$16. f''(x) = 2 + x^3 + x^6 \Rightarrow f'(x) = 2x + \frac{1}{4}x^4 + \frac{1}{7}x^7 + C \Rightarrow f(x) = x^2 + \frac{1}{20}x^5 + \frac{1}{56}x^8 + Cx + D$$

$$17. f''(x) = 1 + x^{4/5} \Rightarrow f'(x) = x + \frac{5}{9}x^{9/5} + C \Rightarrow$$

$$f(x) = \frac{1}{2}x^2 + \frac{5}{9} \cdot \frac{5}{14}x^{14/5} + Cx + D = \frac{1}{2}x^2 + \frac{25}{126}x^{14/5} + Cx + D$$

$$18. f''(x) = \cos x \Rightarrow f'(x) = \sin x + C \Rightarrow f(x) = -\cos x + Cx + D$$

$$19. f'(x) = \sqrt{x}(6+5x) = 6x^{1/2} + 5x^{3/2} \Rightarrow f(x) = 4x^{3/2} + 2x^{5/2} + C.$$

$$f(1) = 6 + C \text{ and } f(1) = 10 \Rightarrow C = 4, \text{ so } f(x) = 4x^{3/2} + 2x^{5/2} + 4.$$

$$20. f'(x) = 2x - 3/x^4 = 2x - 3x^{-4} \Rightarrow f(x) = x^2 + x^{-3} + C \text{ because we're given that } x > 0.$$

$$f(1) = 2 + C \text{ and } f(1) = 3 \Rightarrow C = 1, \text{ so } f(x) = x^2 + 1/x^3 + 1.$$

21. $f'(t) = 2 \cos t + \sec^2 t \Rightarrow f(t) = 2 \sin t + \tan t + C$ because $-\pi/2 < t < \pi/2$.
 $f(\frac{\pi}{3}) = 2(\sqrt{3}/2) + \sqrt{3} + C = 2\sqrt{3} + C$ and $f(\frac{\pi}{3}) = 4 \Rightarrow C = 4 - 2\sqrt{3}$, so $f(t) = 2 \sin t + \tan t + 4 - 2\sqrt{3}$.
22. $f'(x) = 4/\sqrt{1-x^2} \Rightarrow f(x) = 4 \sin^{-1} x + C$. $f(\frac{1}{2}) = 4 \sin^{-1}(\frac{1}{2}) + C = 4 \cdot \frac{\pi}{6} + C$ and $f(\frac{1}{2}) = 1 \Rightarrow \frac{2\pi}{3} + C = 1 \Rightarrow C = 1 - \frac{2\pi}{3}$, so $f(x) = 4 \sin^{-1} x + 1 - \frac{2\pi}{3}$.
23. $f''(x) = 24x^2 + 2x + 10 \Rightarrow f'(x) = 8x^3 + x^2 + 10x + C$. $f'(1) = 8 + 1 + 10 + C$ and $f'(1) = -3 \Rightarrow 19 + C = -3 \Rightarrow C = -22$, so $f'(x) = 8x^3 + x^2 + 10x - 22$ and hence, $f(x) = 2x^4 + \frac{1}{3}x^3 + 5x^2 - 22x + D$.
 $f(1) = 2 + \frac{1}{3} + 5 - 22 + D$ and $f(1) = 5 \Rightarrow D = 22 - \frac{7}{3} = \frac{59}{3}$, so $f(x) = 2x^4 + \frac{1}{3}x^3 + 5x^2 - 22x + \frac{59}{3}$.
24. $f''(x) = 4 - 6x - 40x^3 \Rightarrow f'(x) = 4x - 3x^2 - 10x^4 + C$. $f'(0) = C$ and $f'(0) = 1 \Rightarrow C = 1$, so
 $f'(x) = 4x - 3x^2 - 10x^4 + 1$ and hence, $f(x) = 2x^2 - x^3 - 2x^5 + x + D$. $f(0) = D$ and $f(0) = 2 \Rightarrow D = 2$, so
 $f(x) = 2x^2 - x^3 - 2x^5 + x + 2$.
25. $f''(\theta) = \sin \theta + \cos \theta \Rightarrow f'(\theta) = -\cos \theta + \sin \theta + C$. $f'(0) = -1 + C$ and $f'(0) = 4 \Rightarrow C = 5$, so
 $f'(\theta) = -\cos \theta + \sin \theta + 5$ and hence, $f(\theta) = -\sin \theta - \cos \theta + 5\theta + D$. $f(0) = -1 + D$ and $f(0) = 3 \Rightarrow D = 4$, so
 $f(\theta) = -\sin \theta - \cos \theta + 5\theta + 4$.
26. $f''(t) = 3/\sqrt{t} = 3t^{-1/2} \Rightarrow f'(t) = 6t^{1/2} + C$. $f'(4) = 12 + C$ and $f'(4) = 7 \Rightarrow C = -5$, so $f'(t) = 6t^{1/2} - 5$
and hence, $f(t) = 4t^{3/2} - 5t + D$. $f(4) = 32 - 20 + D$ and $f(4) = 20 \Rightarrow D = 8$, so $f(t) = 4t^{3/2} - 5t + 8$.
27. $f''(x) = x^{-2}, x > 0 \Rightarrow f'(x) = -1/x + C \Rightarrow f(x) = -\ln|x| + Cx + D = -\ln x + Cx + D$ (since $x > 0$).
 $f(1) = 0 \Rightarrow C + D = 0$ and $f(2) = 0 \Rightarrow -\ln 2 + 2C + D = 0 \Rightarrow -\ln 2 + 2C - C = 0$ [since $D = -C$] \Rightarrow
 $-\ln 2 + C = 0 \Rightarrow C = \ln 2$ and $D = -\ln 2$. So $f(x) = -\ln x + (\ln 2)x - \ln 2$.
28. $f''(t) = 2e^t + 3 \sin t \Rightarrow f'(t) = 2e^t - 3 \cos t + C \Rightarrow f(t) = 2e^t - 3 \sin t + Ct + D$.
 $f(0) = 2 + D$ and $f(0) = 0 \Rightarrow D = -2$. $f(\pi) = 2e^\pi + \pi C - 2$ and $f(\pi) = 0 \Rightarrow \pi C = 2 - 2e^\pi \Rightarrow$
 $C = \frac{2 - 2e^\pi}{\pi}$, so $f(t) = 2e^t - 3 \sin t + \frac{2 - 2e^\pi}{\pi}t - 2$.
29. Given $f'(x) = 2x + 1$, we have $f(x) = x^2 + x + C$. Since f passes through $(1, 6)$, $f(1) = 6 \Rightarrow 1^2 + 1 + C = 6 \Rightarrow$
 $C = 4$. Therefore, $f(x) = x^2 + x + 4$ and $f(2) = 2^2 + 2 + 4 = 10$.
30. $f'(x) = x^3 \Rightarrow f(x) = \frac{1}{4}x^4 + C$. $x + y = 0 \Rightarrow y = -x \Rightarrow m = -1$. Now $m = f'(x) \Rightarrow -1 = x^3 \Rightarrow$
 $x = -1 \Rightarrow y = 1$ (from the equation of the tangent line), so $(-1, 1)$ is a point on the graph of f . From f ,
 $1 = \frac{1}{4}(-1)^4 + C \Rightarrow C = \frac{3}{4}$. Therefore, the function is $f(x) = \frac{1}{4}x^4 + \frac{3}{4}$.
31. b is the antiderivative of f . For small x , f is negative, so the graph of its antiderivative must be decreasing. But both a and c are increasing for small x , so only b can be f 's antiderivative. Also, f is positive where b is increasing, which supports our conclusion.

32. We know right away that c cannot be f 's antiderivative, since the slope of c is not zero at the x -value where $f = 0$. Now f is positive when a is increasing and negative when a is decreasing, so a is the antiderivative of f .
33. $v(t) = s'(t) = \sin t - \cos t \Rightarrow s(t) = -\cos t - \sin t + C$. $s(0) = -1 + C$ and $s(0) = 0 \Rightarrow C = 1$, so $s(t) = -\cos t - \sin t + 1$.
34. $v(t) = s'(t) = 1.5\sqrt{t} \Rightarrow s(t) = t^{3/2} + C$. $s(4) = 8 + C$ and $s(4) = 10 \Rightarrow C = 2$, so $s(t) = t^{3/2} + 2$.
35. $a(t) = v'(t) = 10 \sin t + 3 \cos t \Rightarrow v(t) = -10 \cos t + 3 \sin t + C \Rightarrow s(t) = -10 \sin t - 3 \cos t + Ct + D$.
 $s(0) = -3 + D = 0$ and $s(2\pi) = -3 + 2\pi C + D = 12 \Rightarrow D = 3$ and $C = \frac{6}{\pi}$. Thus,
 $s(t) = -10 \sin t - 3 \cos t + \frac{6}{\pi}t + 3$.
36. $a(t) = v'(t) = 10 + 3t - 3t^2 \Rightarrow v(t) = 10t + \frac{3}{2}t^2 - t^3 + C \Rightarrow s(t) = 5t^2 + \frac{1}{2}t^3 - \frac{1}{4}t^4 + Ct + D \Rightarrow$
 $0 = s(0) = D$ and $10 = s(2) = 20 + 4 - 4 + 2C \Rightarrow C = -5$, so $s(t) = -5t + 5t^2 + \frac{1}{2}t^3 - \frac{1}{4}t^4$.
39. By Exercise 38 with $a = -9.8$, $s(t) = -4.9t^2 + v_0t + s_0$ and $v(t) = s'(t) = -9.8t + v_0$. So
 $[v(t)]^2 = (-9.8t + v_0)^2 = (9.8)^2 t^2 - 19.6v_0t + v_0^2 = v_0^2 + 96.04t^2 - 19.6v_0t = v_0^2 - 19.6(-4.9t^2 + v_0t)$.
But $-4.9t^2 + v_0t$ is just $s(t)$ without the s_0 term; that is, $s(t) - s_0$. Thus, $[v(t)]^2 = v_0^2 - 19.6[s(t) - s_0]$.
40. For the first ball, $s_1(t) = -16t^2 + 48t + 432$ from Example 6. For the second ball, $a(t) = -32 \Rightarrow v(t) = -32t + C$,
but $v(1) = -32(1) + C = 24 \Rightarrow C = 56$, so $v(t) = -32t + 56 \Rightarrow s(t) = -16t^2 + 56t + D$, but
 $s(1) = -16(1)^2 + 56(1) + D = 432 \Rightarrow D = 392$, and $s_2(t) = -16t^2 + 56t + 392$. The balls pass each other when
 $s_1(t) = s_2(t) \Rightarrow -16t^2 + 48t + 432 = -16t^2 + 56t + 392 \Leftrightarrow 8t = 40 \Leftrightarrow t = 5$ s.
Another solution: From Exercise 38, we have $s_1(t) = -16t^2 + 48t + 432$ and $s_2(t) = -16t^2 + 24t + 432$.
We now want to solve $s_1(t) = s_2(t - 1) \Rightarrow -16t^2 + 48t + 432 = -16(t - 1)^2 + 24(t - 1) + 432 \Rightarrow$
 $48t = 32t - 16 + 24t - 24 \Rightarrow 40 = 8t \Rightarrow t = 5$ s.
41. Using Exercise 38 with $a = -32$, $v_0 = 0$, and $s_0 = h$ (the height of the cliff), we know that the height at time t is
 $s(t) = -16t^2 + h$. $v(t) = s'(t) = -32t$ and $v(t) = -120 \Rightarrow -32t = -120 \Rightarrow t = 3.75$, so
 $0 = s(3.75) = -16(3.75)^2 + h \Rightarrow h = 16(3.75)^2 = 225$ ft.

42. (a) $EIy'' = mg(L-x) + \frac{1}{2}\rho g(L-x)^2 \Rightarrow EIy' = -\frac{1}{2}mg(L-x)^2 - \frac{1}{6}\rho g(L-x)^3 + C \Rightarrow$
 $EIy = \frac{1}{6}mg(L-x)^3 + \frac{1}{24}\rho g(L-x)^4 + Cx + D$. Since the left end of the board is fixed, we must have $y = y' = 0$
 when $x = 0$. Thus, $0 = -\frac{1}{2}mgL^2 - \frac{1}{6}\rho gL^3 + C$ and $0 = \frac{1}{6}mgL^3 + \frac{1}{24}\rho gL^4 + D$. It follows that
 $EIy = \frac{1}{6}mg(L-x)^3 + \frac{1}{24}\rho g(L-x)^4 + (\frac{1}{2}mgL^2 + \frac{1}{6}\rho gL^3)x - (\frac{1}{6}mgL^3 + \frac{1}{24}\rho gL^4)$ and
 $f(x) = y = \frac{1}{EI} [\frac{1}{6}mg(L-x)^3 + \frac{1}{24}\rho g(L-x)^4 + (\frac{1}{2}mgL^2 + \frac{1}{6}\rho gL^3)x - (\frac{1}{6}mgL^3 + \frac{1}{24}\rho gL^4)]$
- (b) $f(L) < 0$, so the end of the board is a distance approximately $-f(L)$ below the horizontal. From our result in (a), we calculate

$$-f(L) = \frac{-1}{EI} [\frac{1}{2}mgL^3 + \frac{1}{6}\rho gL^4 - \frac{1}{6}mgL^3 - \frac{1}{24}\rho gL^4] = \frac{-1}{EI} (\frac{1}{3}mgL^3 + \frac{1}{8}\rho gL^4) = -\frac{gL^3}{EI} \left(\frac{m}{3} + \frac{\rho L}{8} \right)$$

Note: This is positive because g is negative.

43. Taking the upward direction to be positive we have that for $0 \leq t \leq 10$ (using the subscript 1 to refer to $0 \leq t \leq 10$),
 $a_1(t) = -(9 - 0.9t) = v_1'(t) \Rightarrow v_1(t) = -9t + 0.45t^2 + v_0$, but $v_1(0) = v_0 = -10 \Rightarrow$
 $v_1(t) = -9t + 0.45t^2 - 10 = s_1'(t) \Rightarrow s_1(t) = -\frac{9}{2}t^2 + 0.15t^3 - 10t + s_0$. But $s_1(0) = 500 = s_0 \Rightarrow$
 $s_1(t) = -\frac{9}{2}t^2 + 0.15t^3 - 10t + 500$. $s_1(10) = -450 + 150 - 100 + 500 = 100$, so it takes
 more than 10 seconds for the raindrop to fall. Now for $t > 10$, $a(t) = 0 = v'(t) \Rightarrow$
 $v(t) = \text{constant} = v_1(10) = -9(10) + 0.45(10)^2 - 10 = -55 \Rightarrow v(t) = -55$.
 At 55 m/s, it will take $100/55 \approx 1.8$ s to fall the last 100 m. Hence, the total time is $10 + \frac{100}{55} = \frac{130}{11} \approx 11.8$ s.
44. $v'(t) = a(t) = -22$. The initial velocity is 50 mi/h = $\frac{50 \cdot 5280}{3600} = \frac{220}{3}$ ft/s, so $v(t) = -22t + \frac{220}{3}$. The car stops
 when $v(t) = 0 \Leftrightarrow t = \frac{220}{3 \cdot 22} = \frac{10}{3}$. Since $s(t) = -11t^2 + \frac{220}{3}t$, the distance covered is
 $s(\frac{10}{3}) = -11(\frac{10}{3})^2 + \frac{220}{3} \cdot \frac{10}{3} = \frac{1100}{9} = 122.\bar{2}$ ft.
45. $a(t) = k$, the initial velocity is 30 mi/h = $30 \cdot \frac{5280}{3600} = 44$ ft/s, and the final velocity (after 5 seconds) is
 50 mi/h = $50 \cdot \frac{5280}{3600} = \frac{220}{3}$ ft/s. So $v(t) = kt + C$ and $v(0) = 44 \Rightarrow C = 44$. Thus, $v(t) = kt + 44 \Rightarrow$
 $v(5) = 5k + 44$. But $v(5) = \frac{220}{3}$, so $5k + 44 = \frac{220}{3} \Rightarrow 5k = \frac{88}{3} \Rightarrow k = \frac{88}{15} \approx 5.87$ ft/s².
46. $a(t) = -16 \Rightarrow v(t) = -16t + v_0$ where v_0 is the car's speed (in ft/s) when the brakes were applied. The car stops when
 $-16t + v_0 = 0 \Leftrightarrow t = \frac{1}{16}v_0$. Now $s(t) = \frac{1}{2}(-16)t^2 + v_0t = -8t^2 + v_0t$. The car travels 200 ft in the time that it takes
 to stop, so $s(\frac{1}{16}v_0) = 200 \Rightarrow 200 = -8(\frac{1}{16}v_0)^2 + v_0(\frac{1}{16}v_0) = \frac{1}{32}v_0^2 \Rightarrow v_0^2 = 32 \cdot 200 = 6400 \Rightarrow v_0 = 80$ ft/s
 (54.54 mi/h).

47. Let the acceleration be $a(t) = k \text{ km/h}^2$. We have $v(0) = 100 \text{ km/h}$ and we can take the initial position $s(0)$ to be 0.

We want the time t_f for which $v(t) = 0$ to satisfy $s(t) < 0.08 \text{ km}$. In general, $v'(t) = a(t) = k$, so $v(t) = kt + C$, where $C = v(0) = 100$. Now $s'(t) = v(t) = kt + 100$, so $s(t) = \frac{1}{2}kt^2 + 100t + D$, where $D = s(0) = 0$.

Thus, $s(t) = \frac{1}{2}kt^2 + 100t$. Since $v(t_f) = 0$, we have $kt_f + 100 = 0$ or $t_f = -100/k$, so

$$s(t_f) = \frac{1}{2}k \left(-\frac{100}{k}\right)^2 + 100 \left(-\frac{100}{k}\right) = 10,000 \left(\frac{1}{2k} - \frac{1}{k}\right) = -\frac{5,000}{k}. \text{ The condition } s(t_f) \text{ must satisfy is}$$

$$-\frac{5,000}{k} < 0.08 \Rightarrow -\frac{5,000}{0.08} > k \quad [k \text{ is negative}] \Rightarrow k < -62,500 \text{ km/h}^2, \text{ or equivalently,}$$

$$k < -\frac{3125}{648} \approx -4.82 \text{ m/s}^2. \text{ Thus, a constant deceleration of } 4.82 \text{ m/s}^2 \text{ is required.}$$