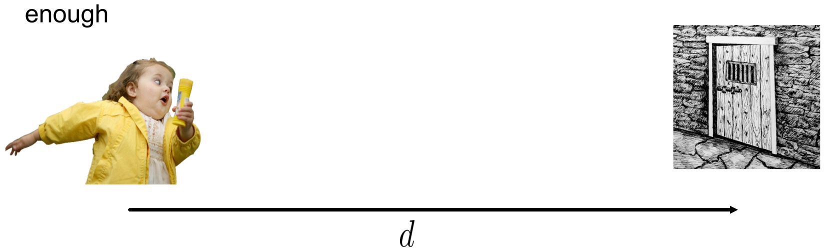
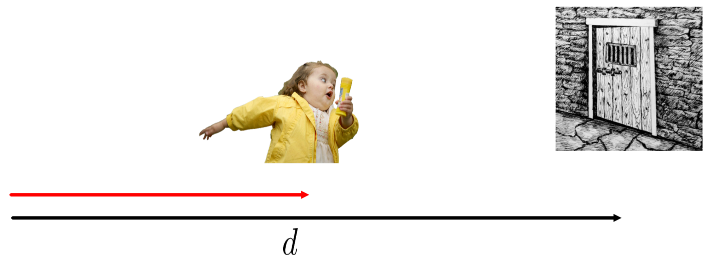
<u>Series</u>

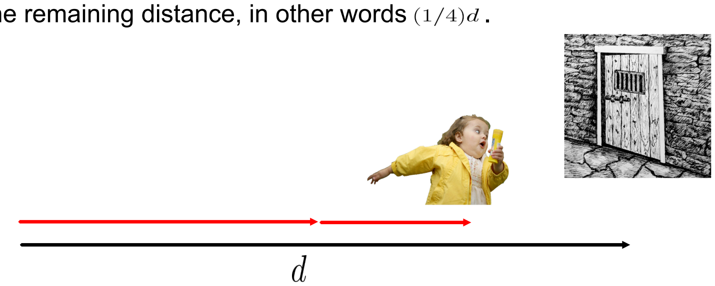
Suppose math class is over and you can't get out of the class room fast enough



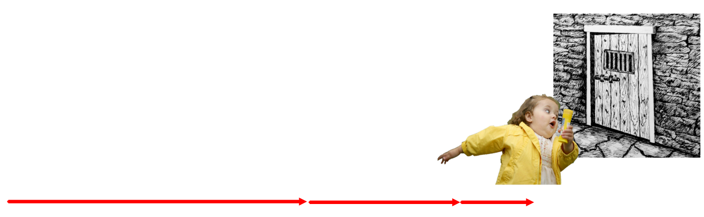
The distance between you and the door is d. Before you get to the door you will have to travel half of the distance d.



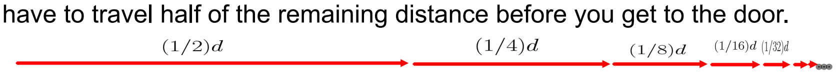
Once you have done that, before you get to the door you will have to travel half of the remaining distance, in other words $(1/4)d$.



You will then, again, have to travel half of the remaining distance which is $(1/8)d$.



In fact, no matter how far you have gone, before you get to the door you will have to travel half of the remaining distance before you get to the door.



Let's make a list of all the distances we have to cover before we can get to the door.

But this is an infinite sequence. There are infinitely many distances in this list. In order to get to the door we must cover infinitely many distances, class will never end!

The good news is, this class was on infinite series and you were paying attention so you know how to get out. It seems like to get out of the class we would need to add up an infinite amount of distances:

$$\frac{1}{2}d+\frac{1}{4}d+\frac{1}{8}d+\frac{1}{16}d+\frac{1}{32}d+\frac{1}{64}d+\ldots$$

Let's start by adding up a finite amount of distances.

These are called the **partial sums**. In general

Notice that they form their own sequence $\{s_n\}$ which may or may not have a limit. In our case

$\{s_1, s_2, s_3, s_4, \ldots\}$

Definition: Given a series

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \ldots$$

Let s_n denote the nth partial sum:

$$s_n = \sum_{i=1}^{n} a_i = a_1 + a_2 + a_3 + \ldots + a_n$$

If the sequence $\{s_n\}$ is convergent and $\lim_{n\to\infty} s_n = s$ exists as a real number, then the series $\sum_{n=1}^{\infty} a_n$ is **convergent** and we write

$$\sum_{n=1}^{\infty} a_n = a_1 + a_2 + a_3 + \ldots = s$$

The number s is called the **sum** of the series. If the sequence $\{s_n\}$ is divergent then the series is called **divergent**.

Example: Determine if the following series converge or diverge. If they converge, find the sum.

1) $\sum_{n=1}^{\infty} \left(\frac{1}{n} - \frac{1}{n+1}\right)$

2) $$\sum_{n=1}^{\infty} \frac{2}{n^2 + 4x + 3}$$

3) $\displaystyle\sum_{n=1}^{\infty} \ln\left(\frac{n+1}{n}\right)$