

Quiz 10

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. (2 marks) §8.1 #8 Find a formula for the general term a_n of the sequence, assuming that the pattern of the first few terms continues.

{5, 8, 11, 14, 17, ...}

$$a_n = 5 + 3(n-1)$$

Question 2. (4 marks) §8.1 #30 Determine whether the sequence converges or diverges. If it converges, find the limit.

$$a_n = \frac{(\ln n)^2}{n}$$

$$\text{Let } f(x) = \frac{(\ln x)^2}{x}$$

$$\lim_{x \rightarrow \infty} f(x)$$

$$= \lim_{x \rightarrow \infty} \frac{(\ln x)^2}{x} \quad \text{i.f. } \frac{\infty}{\infty}$$

$$\stackrel{\hat{H}}{=} \lim_{x \rightarrow \infty} \frac{2 \ln x \cdot \frac{1}{x}}{1}$$

$$\begin{aligned} &= \lim_{x \rightarrow \infty} \frac{2 \ln x}{x} \quad \text{i.f. } \frac{\infty}{\infty} \\ &\stackrel{\hat{H}}{=} \lim_{x \rightarrow \infty} \frac{2(\frac{1}{x})}{1} \\ &= 0 \end{aligned}$$

$$\therefore a_n \rightarrow 0 \quad \text{as } n \rightarrow \infty$$

Question 3. (4 marks) §8.1 #28 Determine whether the sequence converges or diverges. If it converges, find the limit.

$$a_n = \frac{\sin 2n}{1 + \sqrt{n}}$$

$$\frac{-1}{\sqrt{n}} \leq \frac{-1}{1 + \sqrt{n}} \leq \frac{\sin 2n}{1 + \sqrt{n}} \leq \frac{1}{1 + \sqrt{n}} \leq \frac{1}{\sqrt{n}} = c_n$$

\parallel

b_n

$$\lim_{n \rightarrow \infty} \frac{-1}{\sqrt{n}} = 0 = \lim_{n \rightarrow \infty} \frac{1}{\sqrt{n}}$$

\therefore by the squeeze theorem
 $a_n \rightarrow 0$ as $n \rightarrow \infty$.