Dawson College: Calculus II (SCIENCE): 201-NYB-05-S03003: Winter 2013

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Quiz 11

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(5 marks)* §8.2 #28 Determine whether the series is convergent or divergent by expressing s_n as a telescoping sum. If it is convergent, find its sum.

$$\sum_{n=1}^{\infty}\left[e^{1/n}-e^{1/(n+1)}\right]$$

$$S_n = a_1 + a_2 + a_3 + \dots + a_{n-2} + a_{n-1} + a_n$$

$$= \left[e^{1/1} - \cancel{e^{1/2}}\right] + \left[\cancel{e^{1/2}} - \cancel{e^{1/3}}\right] + \left[\cancel{e^{1/3}} - e^{1/4}\right] + \dots +$$

$$\left[e^{1/(n-2)} - \cancel{e^{\frac{1}{n-1}}}\right] + \left[\cancel{e^{1/(n-1)}} - \cancel{e^{1/n}}\right] + \left[\cancel{e^{\frac{1}{n}}} - e^{\frac{1}{n+1}}\right]$$

$$= e - e^{\frac{1}{n+1}}$$

$$S = \lim_{n\to\infty} S_n$$

$$= \lim_{n\to\infty}\left[e - e^{\cancel{\frac{1}{n+1}}\nearrow 0}\right] = e - e^0 = e - 1$$

$\therefore$ the series converges

Question 2. *(5 marks)* §8.3 #19 Determine whether the series is convergent or divergent

$$\sum_{n=2}^{\infty}\frac{1}{n\ln n}$$

Let $f(x) = \dfrac{1}{x\ln x}$

- $f(x)$ is positive for $x > 2$.
- $f(x)$ is continuous for $x > 2$
- $f'(x) = \dfrac{-1}{(x\ln x)^2}\cdot\left[\ln x + x\dfrac{1}{x}\right] < 0$

for $x > 2$ $\therefore$ $f(x)$ is decreasing

$$\int_2^{\infty}\frac{1}{x\ln x}\,dx = \lim_{b\to\infty}\int_2^b \frac{1}{x\ln x}\,dx$$

$u = \ln x$ $\quad$ $u(b) = \ln b$

$du = \frac{1}{x}dx$ $\quad$ $u(2) = \ln 2$

$$= \lim_{b\to\infty}\int_{\ln 2}^{\ln b}\frac{1}{u}\,du$$

$$= \lim_{b\to\infty}\Big[\ln|u|\Big]_{\ln 2}^{\ln b}$$

$$= \lim_{b\to\infty}\left[\ln \cancel{\ln b}^{\nearrow\infty} - \ln\ln 2\right]$$

$\therefore$ integral diverges

$\therefore$ by integral test the series diverges since integral diverges.