Dawson College: **Calculus II** (SCIENCE): 201-NYB-05-S03003: Winter 2013

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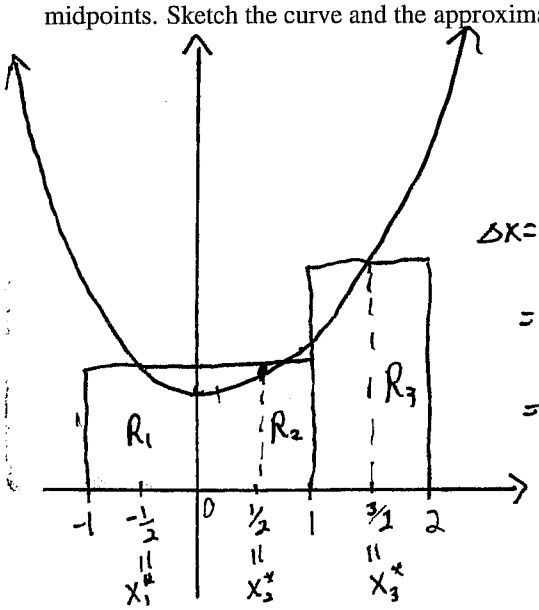
Quiz 3

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(5 marks)* §5.1 #5c Estimate the area under the graph of $f(x) = 1+x^2$ from $x = -1$ to $x = 2$ using three rectangles and the midpoints. Sketch the curve and the approximating rectangles.



$\Delta x = \frac{b-a}{n} = \frac{2-(-1)}{3} = 1$

$\text{Area} \approx R_1 + R_2 + R_3$

$= f(x_1^*)\Delta x + f(x_2^*)\Delta x + f(x_3^*)\Delta x$

$= f\left(\frac{-1}{2}\right)\cdot 1 + f\left(\frac{1}{2}\right)\cdot 1 + f\left(\frac{3}{2}\right)\cdot 1$

$= \left[1+\left(\frac{-1}{2}\right)^2\right]\cdot 1 + \left[1+\left(\frac{1}{2}\right)^2\right]\cdot 1 + \left[1+\left(\frac{3}{2}\right)^2\right]\cdot 1$

$= \left[1+\frac{1}{4}\right] + \left[1+\frac{1}{4}\right] + \left[1+\frac{9}{4}\right]$

$= 3 + \frac{11}{4} = \frac{23}{4}$

Question 2. *(5 marks)* §5.2 #21 Use the form of the definition of the integral given to evaluate the integral.

$\int_{-2}^{0}(x^2+x)\,dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i)\Delta x \qquad \Delta x = \frac{b-a}{n} = \frac{0-(-2)}{n} = \frac{2}{n} \qquad x_i = -2 + i\frac{2}{n}$

$= \lim_{n\to\infty}\sum_{i=1}^{n} f\left(-2+\frac{2i}{n}\right)\frac{2}{n}$

$= \lim_{n\to\infty}\frac{2}{n}\sum_{i=1}^{n}\left[\left(-2+\frac{2i}{n}\right)^2 + \left(-2+\frac{2i}{n}\right)\right]$

$= \lim_{n\to\infty}\frac{2}{n}\sum_{i=1}^{n}\left[4 - 8\frac{i}{n} + 4\frac{i^2}{n^2} - 2 + \frac{2i}{n}\right]$

$= \lim_{n\to\infty}\frac{2}{n}\sum_{i=1}^{n}\left[2 - 6\frac{i}{n} + 4\frac{i^2}{n^2}\right]$

$= \lim_{n\to\infty}\frac{2}{n}\left[\sum_{i=1}^{n} 2 - \sum_{i=1}^{n}\frac{6i}{n} + \sum_{i=1}^{n}\frac{4i^2}{n^2}\right]$

$= \lim_{n\to\infty}\frac{2}{n}\left[\sum_{i=1}^{n} 2 - \frac{6}{n}\sum_{i=1}^{n} i + \frac{4}{n^2}\sum_{i=1}^{n} i^2\right]$

$= \lim_{n\to\infty}\frac{2}{n}\left[2n - \frac{6}{n}\cdot\frac{n(n+1)}{2} + \frac{4}{n^2}\cdot\frac{n(n+1)(2n+1)}{6}\right]$

$= \lim_{n\to\infty}\left[\frac{4n}{n} - \frac{6(n+1)}{n} + \frac{4(n+1)(2n+1)}{3n}\right]$

$= 4 - 6 + \frac{4}{3}\cdot\frac{2}{1}$

$= -2 + \frac{8}{3}$

$= \frac{2}{3}$