

Quiz 5

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. (3 marks) §5.5 #46 Evaluate the definite integral.

$$\int_{-\pi/2}^{\pi/2} \frac{x^2 \sin x}{1+x^6} dx \quad \text{Let } f(x) = \frac{x^2 \sin x}{1+x^6}$$

$$f(-x) = \frac{(-x)^2 \sin(-x)}{1+(-x)^6} = \frac{x^2 (-\sin x)}{1+x^6} \quad \text{since } \sin x \text{ is odd}$$

$$= -\frac{x^2 \sin x}{1+x^6} = -f(x) \quad \text{so } f(x) \text{ is odd}$$

$$\int_{-\pi/2}^{\pi/2} \frac{x^2 \sin x}{1+x^6} dx = 0$$

Question 2. (3 marks) §5.5 #14 Evaluate the indefinite integral.

$$\int \frac{dt}{\cos^2 t \sqrt{1+\tan t}} = \int \frac{\sec^2 t dt}{\sqrt{1+\tan t}} = \int \frac{1}{\sqrt{u}} du$$

$$u = 1+\tan t$$

$$du = \sec^2 t dt$$

$$= \int u^{-1/2} du$$

$$= 2\sqrt{u} + C$$

$$= 2\sqrt{1+\tan t} + C$$

Question 3. (4 marks) §5.5 #51

$$\int_e^{e^4} \frac{dx}{x\sqrt{\ln x}} = \int_1^4 \frac{1}{\sqrt{u}} du = \left[2\sqrt{u} \right]_1^4$$

$$u = \ln x$$

$$du = \frac{1}{x} dx$$

$$u(e^4) = \ln e^4 = 4$$

$$u(e) = \ln e = 1$$

$$= 2\sqrt{4} - 2\sqrt{1}$$

$$= 4 - 2$$

$$= 2$$