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Quiz 6

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(5 marks)* §6.1 #12 Evaluate the indefinite integral.

$\int \sin^{-1} x\,dx = uv - \int v\,du$ $\quad u = \arcsin x$ $\quad du = \frac{1}{\sqrt{1-x^2}}\,dx$

$= x\arcsin x - \int \frac{x}{\sqrt{1-x^2}}\,dx$ $\quad v = x$ $\quad dv = dx$

$u = 1-x^2$

$du = -2x\,dx$

$\frac{du}{-2} = x\,dx$

$= x\arcsin x - \int \frac{1}{\sqrt{u}} \frac{du}{-2}$

$= x\arcsin x + \frac{1}{2}\cdot 2\sqrt{u} + C$

$= x\arcsin x + \sqrt{1-x^2} + C$

Question 2. *(5 marks)* §6.2 #30 Evaluate the definite integral.

$\int_{\pi/4}^{\pi/2} \cot^3 x\,dx = \int_{\pi/4}^{\pi/2} \cot^2 x \cot x\,dx$

$= \int_{\pi/4}^{\pi/2} (\csc^2 x - 1)\cot x\,dx$

$= \int_{\pi/4}^{\pi/2} \csc^2 x \cot x - \cot x\,dx$

$= \int_{\pi/4}^{\pi/2} \csc^2 x \cot x\,dx - \int_{\pi/4}^{\pi/2} \cot x\,dx$

$u = \cot x$ $\quad u(\pi/2) = \cot\frac{\pi}{2} = 0$

$du = -\csc^2 x\,dx$ $\quad u(\pi/4) = \cot\frac{\pi}{4} = 1$

$-du = \csc^2 x\,dx$

$= \int_1^0 u(-du) - \left[\ln|\sin x|\right]_{\pi/4}^{\pi/2}$

$= \left[-\frac{u^2}{2}\right]_1^0 - \underbrace{\ln\left|\sin\frac{\pi}{2}\right|}_{0} + \ln\left|\sin\frac{\pi}{4}\right|$

$= \frac{1}{2} + \ln\left(\frac{1}{\sqrt{2}}\right)$

Question 3. *(5 marks)* §6.2 #49 Evaluate the indefinite integral.

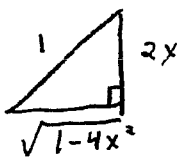
$$\int \sqrt{1-4x^2}\,dx = \int \sqrt{1-(2x)^2}\,dx \qquad 2x = \sin\theta \quad x = \tfrac{1}{2}\sin\theta$$

$$= \int \sqrt{1-\sin^2\theta}\,\tfrac{1}{2}\cos\theta\,d\theta \qquad dx = \tfrac{1}{2}\cos\theta\,d\theta$$

$$= \tfrac{1}{2}\int \sqrt{\cos^2\theta}\,\cos\theta\,d\theta \qquad \frac{opp}{hyp} = \frac{2x}{1} = \sin\theta \qquad \theta = \arcsin 2x$$

$$= \tfrac{1}{2}\int \cos^2\theta\,d\theta$$

$$= \tfrac{1}{2}\int \frac{1+\cos 2\theta}{2}\,d\theta$$



$$= \tfrac{1}{4}\int 1+\cos 2\theta\,d\theta$$

$$= \tfrac{1}{4}\left[\theta + \frac{\sin 2\theta}{2}\right] + C$$

$$= \tfrac{1}{4}\left[\theta + \frac{2\sin\theta\cos\theta}{2}\right] + C$$

$$= \tfrac{1}{4}\arcsin(2x) + \tfrac{1}{4\cdot 2}\cdot 2x\cdot\sqrt{1-4x^2} + C$$

$$= \tfrac{1}{4}\arcsin(2x) + \tfrac{1}{2}x\sqrt{1-4x^2} + C$$