Dawson College: **Calculus II** (SCIENCE): 201-NYB-05-S03003: Winter 2013

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Quiz 7

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(5 marks)* §6.3 #20 Evaluate the indefinite integral.

$$\int \frac{x^2-5x+16}{(2x+1)(x-2)^2}\,dx = \int \frac{3}{2x+1} - \frac{1}{x-2} + \frac{2}{(x-2)^2}\,dx = \frac{3}{2}\ln|2x+1| - \ln|x-2| - \frac{2}{x-2} + C$$

$$\frac{x^2-5x+16}{(2x+1)(x-2)^2} = \frac{A_1}{(2x+1)} + \frac{A_2}{(x-2)} + \frac{A_3}{(x-2)^2}$$

$$\frac{(x^2-5x+16)\cancel{(2x+1)(x-2)^2}}{\cancel{(2x+1)(x-2)^2}} = \frac{A_1\cancel{(2x+1)}(x-2)^2}{\cancel{(2x+1)}} + \frac{A_2(2x+1)(x-2)^{\cancel{2}}}{\cancel{(x-2)}} + \frac{A_3(2x+1)\cancel{(x-2)^2}}{\cancel{(x-2)^2}}$$

$$x^2-5x+16 = A_1(x-2)^2 + A_2(2x+1)(x-2) + A_3(2x+1)$$

Let $x=2$: $(2)^2-5(2)+16 = A_1(2-2)^2 + A_2(2(2)+1)(2-2) + A_3(2(2)+1)$

$$10 = 5A_3 \Leftrightarrow A_3 = 2$$

Let $x=\frac{-1}{2}$: $(-\frac{1}{2})^2-5(-\frac{1}{2})+16 = A_1(-\frac{1}{2}-2)^2 + A_2(2(-\frac{1}{2})+1)(-\frac{1}{2}-2) + A_3(2(-\frac{1}{2})+1)$

$$\frac{75}{4} = A_1\left(\frac{25}{4}\right) \Leftrightarrow A_1 = 3$$

Let $x=0$: $0^2-5(0)+16 = 3(0-2)^2 + A_2(2\cdot0+1)(0-2) + A_3(2(0)+1)$

$$16 = 12 + (-2A_2) + 2 \Leftrightarrow A_2 = -1$$

Question 2. *(5 marks)* §6.6 #32 Evaluate the definite integral.

$$\int_0^1 \frac{\ln x}{\sqrt{x}}\,dx = \lim_{a\to0^+}\int_a^1 \frac{\ln x}{\sqrt{x}}\,dx = \lim_{a\to0^+}\left[[uv]_a^1 - \int_a^1 u\,dv \right]$$

infinite discontinuity at $x=0$

$u = \ln x \quad du = \frac{1}{x}dx$

$v = 2\sqrt{x} \quad dv = \frac{1}{\sqrt{x}}dx$

$$= \lim_{a\to0^+}\left[\left[2\sqrt{x}\ln x\right]_a^1 - \int_a^1 2\sqrt{x}\,\frac{1}{x}\,dx \right]$$

$$= \lim_{a\to0^+}\left[\underbrace{2\sqrt{1}\ln 1}_{0} \overbrace{-2\sqrt{a}\ln a}^{\text{i.f. } 0\cdot-\infty} - 2\int_a^1 \frac{1}{\sqrt{x}}\,dx \right]$$

i.f. $\frac{-\infty}{\infty}$

$$= \lim_{a\to0^+}\left[\frac{-2\ln a}{\frac{1}{\sqrt{a}}} - 2\left[2\sqrt{x}\right]_a^1 \right]$$

$$= \lim_{a\to0^+}\left[\frac{-2/a}{\frac{-1}{2a^{3/2}}} - 4\sqrt{1} + 4\sqrt{a} \right]$$

$$= -4 + \lim_{a\to0^+} \frac{4\cancel{a^{3/2}}\,a^{1/2}}{\cancel{a}} = -4$$