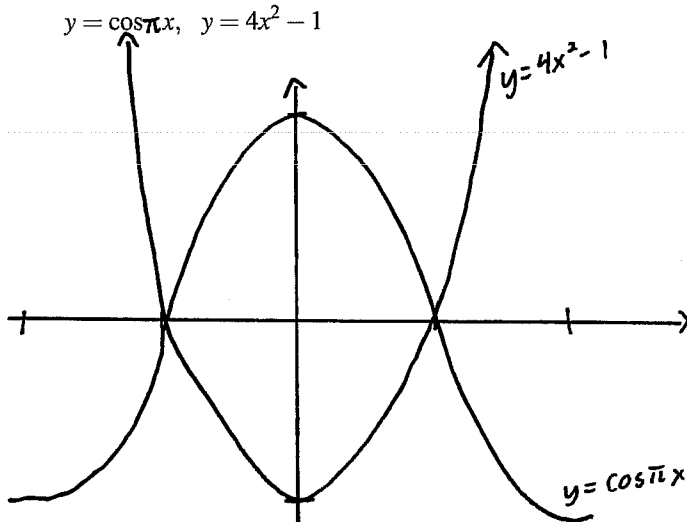Name: Y. Lamontagne
Student ID: _____

Quiz 8

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(5 marks)* §7.1 #20 Sketch the region enclosed by the given curves and find its area.

$y = \cos \pi x, \quad y = 4x^2 - 1$

$$Area = \int_{-\frac{1}{2}}^{\frac{1}{2}} \cos \pi x - [4x^2 - 1] \, dx$$

$$= \left[\frac{\sin \pi x}{\pi} - \frac{4x^3}{3} + x \right]_{-\frac{1}{2}}^{\frac{1}{2}}$$

$$= \left[\frac{\sin \pi(\frac{1}{2})}{\pi} - \frac{4(\frac{1}{2})^2}{3} + \frac{1}{2} \right]$$

$$- \left[\frac{\sin \pi(-\frac{1}{2})}{\pi} - \frac{4(-\frac{1}{2})^2}{3} - \frac{1}{2} \right]$$

$$= \frac{1}{\pi} - \frac{1}{3\cdot 2} + \frac{1}{2} + \frac{1}{\pi} - \frac{1}{3\cdot 2} + \frac{1}{2}$$

$$= \frac{2}{\pi} + \frac{2}{3}$$

Question 2. *(5 marks)* §7.4 #15 Find the exact length of the curve.

$y = \frac{1}{4}x^2 - \frac{1}{2}\ln x, \quad 1 \le x \le 2$

$$S = \int_1^2 \sqrt{1 + (y')^2} \, dx$$

$$y' = \frac{1}{2}x - \frac{1}{2}\frac{1}{x}$$

$$= \int_1^2 \sqrt{1 + \left(\frac{x}{2} - \frac{1}{2x}\right)^2} \, dx = \int_1^2 \sqrt{1 + \frac{x^2}{4} - \frac{1}{2} + \frac{1}{4x^2}} \, dx$$

$$= \int_1^2 \sqrt{\frac{x^2}{4} + \frac{1}{2} + \frac{1}{4x^2}} \, dx$$

$$= \int_1^2 \sqrt{\left(\frac{x}{2} + \frac{1}{2x}\right)^2} \, dx$$

$$= \int_1^2 \left| \frac{x}{2} + \frac{1}{2x} \right| \, dx$$

$$= \int_1^2 \frac{x}{2} + \frac{1}{2x} \, dx$$

$$= \left[\frac{x^2}{4} + \frac{1}{2}\ln|x| \right]_1^2$$

$$= \frac{2^2}{4} + \frac{1}{2}\ln 2 - \frac{1}{4} - \frac{1}{2}\ln 1$$

$$= \frac{3}{4} + \ln\sqrt{2}$$