

Name: _____
Student ID: _____

Test 2

This test is graded out of 45 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. (5 marks) Evaluate the definite integral

$$\int_0^{\sqrt[3]{\pi/4}} x^2 \tan^2 x^3 \, dx$$

Question 2. (5 marks) Evaluate the indefinite integral

$$\int \frac{x^3 + x^2 + 1}{x^3 + x^2 - 6x} dx$$

Question 3. (5 marks) Find the exact length of the curve

$$y = \ln(\cos x), \quad 0 \leq x \leq \frac{\pi}{3}$$

Question 4. (5 marks) Sketch the region(s) enclosed by the given curves and set up the integral to find the total area of the region(s).

$$y = x, y = \sqrt{x}, y = -x^2$$

Question 5. (5 marks) Determine whether the integral is convergent or divergent. Evaluate the integral if convergent.

$$\int_{2\sqrt{3}}^{\infty} \frac{1}{x\sqrt{x^2-3}} dx$$

Question 6. (5 marks) Evaluate the definite integral

$$\int_3^5 \sqrt{x^2 - 2x - 3} \, dx$$

given that

$$\int_0^{\pi/3} \sec^3 x \, dx = \sqrt{3} + \frac{1}{2} \ln(2 + \sqrt{3}).$$

Question 7. (5 marks) Evaluate the indefinite integral

$$\int \arctan(\sqrt{x}) \, dx$$

Question 8. (5 marks) Find the value(s) of the constant p for which the integral

$$\int_0^1 \frac{1}{x^p} dx$$

converges. Evaluate the integral for this value(s) of p .

Question 9. (5 marks) Evaluate the definite integral

$$\int_0^{\frac{\pi}{4}} e^{\cos x} \sin 2x \, dx$$

Bonus Question. (3 marks) Find the exact length of the curve

$$y = \int_2^{\frac{e^x+1}{e^x-1}} \frac{1}{y} dy \quad [\ln 2, \ln 3]$$