

Name: _____
Student ID: _____

Test 3

This test is graded out of 45 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. (5 marks) Set up the integral to find the volume of the solid obtained from the region in the first quadrant bounded by the graphs of $y = \cos x$, $y = \frac{4x}{\pi\sqrt{2}}$ and $x = 0$ rotated about the line $x = \frac{\pi}{4}$.

Question 2. (5 marks) Set up the integral to find the volume of the solid obtained from the region bounded by the graphs of $x - y = 1$, $y = x^2 - 4x + 3$ rotated about the line $y = 3$.

Question 3. (5 marks)

- a. (2 marks) Find a formula for the general term a_n of the sequence, assuming that the pattern of the first few terms continues.

$$\left\{ \tan(1), 2 \tan\left(\frac{1}{2}\right), 3 \tan\left(\frac{1}{3}\right), 4 \tan\left(\frac{1}{4}\right), 5 \tan\left(\frac{1}{5}\right), \dots \right\}_{n=1}^{\infty}$$

- b. (3 marks) Determine the limit of a_n as $n \rightarrow \infty$.

Question 4. (5 marks) Determine whether the series is convergent or divergent. If it is convergent find its sum.

$$\sum_{n=2}^{\infty} \left[e^{1/n} - e^{1/(n+1)} \right]$$

Question 5. (5 marks) Determine whether the series is convergent or divergent. If it is convergent find its sum.

$$\sum_{n=2}^{\infty} \frac{2^n - 3^{n-1}}{4^{n+1}}$$

Question 6. (5 marks) Determine whether the series is convergent or divergent. If it is convergent find its sum.

$$\sum_{n=3}^{\infty} \frac{3n^2}{n(n+3)}$$

Question 7. (5 marks) Determine whether the series is convergent or divergent.

$$\sum_{n=0}^{\infty} \frac{1 + \sin n}{10^n}$$

Question 8. (5 marks) Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

$$\sum_{n=3}^{\infty} \frac{(-1)^n}{n \ln n}$$

Question 9. (5 marks) Determine whether the series is absolutely convergent, conditionally convergent, or divergent.

$$\sum_{n=4}^{\infty} \frac{(-1)^n 2^{n^2}}{n!}$$

Bonus Question. (3 marks) Let $\{b_n\}$ be a sequence of positive numbers that converge to $\frac{1}{2}$. Determine whether the given series is absolutely convergent

$$\sum_{n=1}^{\infty} \frac{(-1)^n n!}{n^n b_1 b_2 b_3 \dots b_n}$$