Dawson College: Calculus II (SCIENCE): 201-NYB-05-S09: Winter 2013

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Quiz 10

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(2 marks)* §8.1 #7 Find a formula for the general term a_n of the sequence, assuming that the pattern of the first few terms continues.

$$\left\{\frac{1}{2}, -\frac{4}{3}, \frac{9}{4}, -\frac{16}{5}, \frac{25}{6}, \cdots\right\} \qquad a_n = (-1)^{n+1}\frac{n^2}{n+1}$$

Question 2. *(4 marks)* §8.1 #30 Determine whether the sequence converges or diverges. If it converges, find the limit.

$$a_n = \frac{(\ln n)^2}{n}$$

Let $f(x) = \frac{(\ln x)^2}{x}$

$$\lim_{x\to\infty} f(x)$$

$$= \lim_{x\to\infty} \frac{(\ln x)^2}{x} \quad \text{l.f. } \frac{\infty}{\infty}$$

$$\overset{\hat{H}}{=} \lim_{x\to\infty} \frac{2\ln x \cdot \frac{1}{x}}{1}$$

$$= \lim_{x\to\infty} \frac{2\ln x}{x} \quad \text{l.f. } \frac{\infty}{\infty}$$

$$\overset{\hat{H}}{=} \lim_{x\to\infty} \frac{2\frac{1}{x}}{1}$$

$$= 0$$

$\therefore a_n \to 0$ as $n \to \infty$

Question 3. *(4 marks)* §8.1 #31 Determine whether the sequence converges or diverges. If it converges, find the limit.

$$a_n = \ln(2n^2+1) - \ln(n^2+1) = \ln\left(\frac{2n^2+1}{n^2+1}\right)$$

Let $f(x) = \ln\left(\frac{2x^2+1}{x^2+1}\right)$

$$\lim_{x\to\infty} f(x)$$

$$= \lim_{x\to\infty} \ln\left(\frac{2x^2+1}{x^2+1}\right)$$

$$= \ln\left(\lim_{x\to\infty} \frac{2x^2+1}{x^2+1}\right)$$

$$= \ln(2)$$

$\therefore a_n \to \ln 2$ as $n \to \infty$