

Quiz 10

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. (2 marks) §8.1 #5 Find a formula for the general term a_n of the sequence, assuming that the pattern of the first few terms continues.

$$\left\{-3, 2, -\frac{4}{3}, \frac{8}{9}, -\frac{16}{27}, \dots\right\} \quad a_n = (-1)^n \frac{2^{n-1}}{3^{n-2}}$$

Question 2. (4 marks) §8.1 #24 Determine whether the sequence converges or diverges. If it converges, find the limit.

$$a_n = \ln(n+1) - \ln n = \ln\left(\frac{n+1}{n}\right)$$

$$\text{Let } f(x) = \ln\left(\frac{x+1}{x}\right) \quad \therefore a_n \rightarrow 0 \text{ as } n \rightarrow \infty$$

$$\lim_{x \rightarrow \infty} f(x)$$

$$= \lim_{x \rightarrow \infty} \ln\left(\frac{x+1}{x}\right)$$

$$= \ln\left(\lim_{x \rightarrow \infty} \frac{x+1}{x}\right) = \ln(1) = 0$$

Question 3. (4 marks) §8.1 #24 Determine whether the sequence converges or diverges. If it converges, find the limit.

$$\{n^2 e^{-n}\}$$

$$\text{Let } f(x) = x^2 e^{-x} = \frac{x^2}{e^x}$$

$$\lim_{x \rightarrow \infty} f(x)$$

$$= \lim_{x \rightarrow \infty} \frac{x^2}{e^x} \quad \text{l.f. } \frac{\infty}{\infty}$$

$$\stackrel{\text{H}}{=} \lim_{x \rightarrow \infty} \frac{2x}{e^x} \quad \text{l.f. } \frac{\infty}{\infty}$$

$$\stackrel{\text{H}}{=} \lim_{x \rightarrow \infty} \frac{2}{e^x} = 0$$

$$\therefore \{n^2 e^{-n}\} \rightarrow 0 \text{ as } n \rightarrow \infty$$