

Quiz 11

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. (5 marks) §8.2 #40 If the n th partial sum of a series $\sum_{n=1}^{\infty} a_n$ is

$$s_n = 3 - n2^{-n}$$

$$S_n = a_1 + a_2 + a_3 + \dots + a_{n-1} + a_n$$

find a_n and $\sum_{n=1}^{\infty} a_n$.

$$S_{n-1} = a_1 + a_2 + a_3 + \dots + a_{n-1}$$

$$a_n = S_n - S_{n-1} = 3 - n2^{-n} - [3 - (n-1)2^{-(n-1)}] = \frac{(n-1)}{2^{n-1}} - \frac{n}{2^n}$$

$$\begin{aligned} S &= \lim_{n \rightarrow \infty} S_n \\ &= \lim_{n \rightarrow \infty} \left[3 - \frac{n}{2^n} \right] \quad \text{lit. } \frac{\infty}{\infty} \\ &= \lim_{n \rightarrow \infty} \left[3 - \frac{1}{2^n \ln 2} \right] \quad n \in \mathbb{R} \\ &= 3 \end{aligned}$$

Question 2. (4 marks) §8.3 #28 Determine whether the series is convergent or divergent

$$\sum_{n=1}^{\infty} \frac{1 + \sin n}{10^n}$$

$$\text{Let } a_n = \frac{1 + \sin n}{10^n}$$

$$a_n = \frac{1 + \sin n}{10^n} \leq \frac{1 + 1}{10^n} = \frac{2}{10^n} = 2 \left(\frac{1}{10} \right)^n = b_n$$

$\sum b_n$ converges since geometric series where $r = \frac{1}{10} < 1$.

By the comparison test the series $\sum a_n$ converges.