Dawson College: Calculus II (SCIENCE): 201-NYB-05-S09: Winter 2013

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Quiz 12

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(5 marks)* §8.5 #16 Find the radius of convergence and interval of convergence of the series.

$$\sum_{n=0}^{\infty}(-1)^n\frac{(x-3)^n}{2n+1}$$

Let $a_n(x) = (-1)^n\frac{(x-3)^n}{2n+1}$ and by the ratio test

$$\lim_{n\to\infty}\left|\frac{a_{n+1}(x)}{a_n(x)}\right| = \lim_{n\to\infty}\left|\frac{(-1)^{n+1}\frac{(x-3)^{n+1}}{2(n+1)+1}}{(-1)^n\frac{(x-3)^n}{2n+1}}\right| = \lim_{n\to\infty}\left|(x-3)\frac{2n+1}{2n+3}\right|$$

$$= |x-3|\lim_{n\to\infty}\frac{2n+1}{2n+3}$$

$$= |x-3| < 1 = R$$

$\therefore$ radius of convergence is $R=1$. and

$$|x-3|<1$$
$$-1<x-3<1$$
$$2<x<4$$

To find the interval of convergence lets test the endpoints $x=2$ and $x=4$

$x=2$

$$\sum_{n=0}^{\infty}\frac{(-1)^n(2-3)^n}{2n+1} = \sum_{n=0}^{\infty}\frac{(-1)^n(-1)^n}{2n+1} = \sum_{n=0}^{\infty}\underbrace{\frac{1}{2n+1}}_{a_n}$$

Let $\sum b_n = \sum \frac{1}{n}$ div. since p-series where $p=1$ and

$$\lim_{n\to\infty}\frac{a_n}{b_n} = \lim_{n\to\infty}\frac{\frac{1}{2n+1}}{\frac{1}{n}} = \lim_{n\to\infty}\frac{n}{2n+1} = \frac{1}{2} > 0$$

$\therefore$ does not converge at $x=2$ by limit comp. test

$x=4$

$$\sum_{n=0}^{\infty}\frac{(-1)^n(4-3)^n}{2n+1} = \sum_{n=0}^{\infty}\frac{(-1)^n}{2n+1}$$

where $b_n = \frac{1}{2n+1}$

① $b_{n+1} \overset{?}{\leq} b_n$

$$\frac{1}{2n+3} \overset{?}{\leq} \frac{1}{2n+1}$$

$$2n+1 \overset{?}{\leq} 2n+3$$

$$0 \overset{?}{\leq} 2$$

② $\lim_{n\to\infty} b_n = 0$

$\therefore$ converges at $x=4$ by A.S.T.

$\therefore$ interval of convergence $(2,4]$

Question 2. *(5 marks)* §8.7 #16 Find the Taylor series for $f(x)$ centered at the given value of a. [Assume that f has a power series expansion. Do not show that $R_n(x) \to 0$.]

$f(x) = \sin x, \quad a = \pi/2$

$$f(x) = \sin x \qquad f(\tfrac{\pi}{2}) = \sin\tfrac{\pi}{2} = 1$$
$$f'(x) = \cos x \qquad f'(\tfrac{\pi}{2}) = \cos\tfrac{\pi}{2} = 0$$
$$f''(x) = -\sin x \qquad f''(\tfrac{\pi}{2}) = -\sin\tfrac{\pi}{2} = -1$$
$$f'''(x) = -\cos x \qquad f'''(\tfrac{\pi}{2}) = -\cos\tfrac{\pi}{2} = 0$$
$$f^{(4)}(x) = \sin x \qquad f^{(4)}(\tfrac{\pi}{2}) = \sin\tfrac{\pi}{2} = 1$$
$$f^{(5)}(x) = \cos x \qquad f^{(5)}(\tfrac{\pi}{2}) = \cos\tfrac{\pi}{2} = 0$$

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(\frac{\pi}{2})(x-\frac{\pi}{2})^n}{n!}$$

$$= f(\tfrac{\pi}{2}) + \frac{f'(\frac{\pi}{2})(x-\frac{\pi}{2})}{1!} + \frac{f''(\frac{\pi}{2})(x-\frac{\pi}{2})^2}{2!} + \frac{f'''(\frac{\pi}{2})(x-\frac{\pi}{2})^3}{3!}$$
$$+ \frac{f^{(4)}(\frac{\pi}{2})(x-\frac{\pi}{2})^4}{4!} + \cdots$$

$$= 1 - \frac{(x-\frac{\pi}{2})^2}{2!} + \frac{(x-\frac{\pi}{2})^2}{4!} - \frac{(x-\pi/2)^6}{6!} + \cdots$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^n (x-\frac{\pi}{2})^{2n}}{(2n)!}$$