Dawson College: Calculus II (SCIENCE): 201-NYB-05-S09: Winter 2013

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Quiz 12

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(5 marks)* §8.2 #20 Find the radius of convergence and interval of convergence of the series.

$$\sum_{n=1}^{\infty} \frac{(2x-1)^n}{5^n\sqrt{n}}$$

Let $a_n(x) = \frac{(2x-1)^n}{5^n\sqrt{n}}$ and by the ratio test

$$\lim_{n\to\infty}\left|\frac{a_{n+1}(x)}{a_n(x)}\right| = \lim_{n\to\infty}\left|\frac{\frac{(2x-1)^{n+1}}{5^{n+1}\sqrt{n+1}}}{\frac{(2x-1)^n}{5^n\sqrt{n}}}\right| = \lim_{n\to\infty}\left|\frac{(2x-1)^{n+1}}{5^{n+1}\sqrt{n+1}}\cdot\frac{5^n\sqrt{n}}{(2x-1)^n}\right|$$

$$= \lim_{n\to\infty}\left|\frac{(2x-1)}{5}\right|\cdot\frac{\sqrt{n}}{\sqrt{n+1}}$$

$$= \left|\frac{(2x-1)}{5}\right| < 1$$

$$|(2x-1)| < 5$$

$$|(x-\tfrac{1}{2})| < \tfrac{5}{2} = R$$

$\therefore$ the radius of convergence is $R = \frac{5}{2}$ and $|x-\frac{1}{2}| < \frac{5}{2}$

$$-\frac{5}{2} < x - \frac{1}{2} < \frac{5}{2}$$

$$-2 < x < 3$$

To find the interval of convergence lets test the end points $x=-2$ and $x=3$

$x = -2$: $\sum_{n=0}^{\infty}\frac{(2(-2)-1)^n}{5^n\sqrt{n}} = \sum_{n=0}^{\infty}\frac{(-5)^n}{5^n\sqrt{n}} = \sum_{n=0}^{\infty}\frac{(-1)^n}{\sqrt{n}}$ where $b_n = \frac{1}{\sqrt{n}}$

① $b_{n+1} \overset{?}{\leq} b_n$

$\frac{1}{\sqrt{n+1}} \overset{?}{\leq} \frac{1}{\sqrt{n}}$

$\sqrt{n} \overset{?}{\leq} \sqrt{n+1}$

$n \overset{?}{\leq} n+1$

$0 \leq 1$

② $\lim_{n\to\infty} b_n = \lim_{n\to\infty}\frac{1}{\sqrt{n}} = 0$

$\therefore$ converges at $x =$ by A.S.T.

$x = 3$: $\sum_{n=0}^{\infty}\frac{(2(3)-1)^n}{5^n\sqrt{n}} = \sum_{n=0}^{\infty}\frac{(5)^n}{5^n\sqrt{n}} = \sum_{n=0}^{\infty}\frac{1}{\sqrt{n}}$ divergent since p-series where $p = \frac{1}{2} < 1$

$\therefore$ does not converge at $x = 3$

$\therefore$ interval of convergence is $[-2, 3)$

Question 2. *(5 marks)* §8.7 #17 Find the Taylor series for $f(x)$ centered at the given value of a. [Assume that f has a power series expansion. Do not show that $R_n(x) \to 0$.]

$f(x) = \cos x, \quad a = \pi$

$f(x) = \cos x \qquad f(\pi) = \cos \pi = -1$

$f'(x) = -\sin x \qquad f'(\pi) = -\sin \pi = 0$

$f''(x) = -\cos x \qquad f''(\pi) = -\cos \pi = 1$

$f'''(x) = \sin x \qquad f'''(\pi) = \sin \pi = 0$

$f^{(4)}(x) = \cos x \qquad f^{(4)}(\pi) = \cos \pi = -1$

$f^{(5)}(x) = -\sin x \qquad f^{(5)}(\pi) = -\sin \pi = 0$

$$f(x) = \sum_{n=0}^{\infty} \frac{f^{(n)}(\pi)(x-\pi)^n}{n!} = f(\pi) + f'(\pi)(x-\pi) + \frac{f''(\pi)(x-\pi)^2}{2!} + \frac{f'''(\pi)(x-\pi)^3}{3!}$$

$$+ \frac{f^{(4)}(\pi)(x-\pi)^4}{4!} + \dots$$

$$= -1 + \frac{1}{2!}(x-\pi)^2 - \frac{(x-\pi)^4}{4!}$$

$$= \sum_{n=0}^{\infty} \frac{(-1)^{n+1}(x-\pi)^{2n}}{(2n)!}$$