

Quiz 5

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. (3 marks) §5.5 #49 Evaluate the definite integral.

$$\int_{-\pi/4}^{\pi/4} \underbrace{(x^3 + x^4 \tan x)}_{f(x)} dx = 0$$

$$\begin{aligned} f(-x) &= (-x)^3 + (-x)^4 \tan(-x) \\ &= -x^3 + x^4 (-\tan x) \\ &= -(x^3 + x^4 \tan x) \\ &= -f(x) \end{aligned}$$

∴ ∴ $f(x)$ is odd

Question 2. (3 marks) §5.5 #36 Evaluate the indefinite integral.

$$\begin{aligned} \int \frac{x}{1+x^4} dx &= \int \frac{x}{1+(x^2)^2} dx = \int \frac{1}{1+u^2} \frac{du}{2} \\ u &= x^2 & &= \frac{1}{2} \arctan u + C \\ du &= 2x dx & &= \frac{1}{2} \arctan x^2 + C \\ \frac{du}{2} &= x dx \end{aligned}$$

Question 3. (4 marks) §5.5 #42 Evaluate the definite integral.

$$\begin{aligned} \int_{1/6}^{1/2} \csc \pi t \cot \pi t dt &= \int_{\pi/6}^{\pi/2} \csc u \cot u \frac{du}{\pi} = \frac{1}{\pi} \left[-\csc u \right]_{\pi/6}^{\pi/2} \\ &= \frac{1}{\pi} \left[\csc \frac{\pi}{6} - \csc \frac{\pi}{2} \right] \\ &= \frac{1}{\pi} \left[2 - 1 \right] = \frac{1}{\pi} \end{aligned}$$

$$\begin{aligned} u &= \pi t \\ du &= \pi dt \\ \frac{du}{\pi} &= dt \\ u(1/2) &= \frac{\pi}{2} \\ u(1/6) &= \frac{\pi}{6} \end{aligned}$$