

Quiz 5

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. (3 marks) §5.5 #54 Find the average value of the function on the given interval.

$$f(x) = \sin 4x, \quad [-\pi, \pi]$$

$$\begin{aligned} f_{\text{ave}} &= \frac{1}{b-a} \int_a^b f(x) dx \\ &= \frac{1}{\pi - (-\pi)} \int_{-\pi}^{\pi} \underbrace{\sin 4x dx}_{f(x)} \\ &= 0 \end{aligned}$$

$$\begin{aligned} f(-x) &= \sin(-4x) = -\sin(4x) = -f(x) \\ \therefore f(x) &\text{ is odd} \end{aligned}$$

Question 2. (3 marks) §5.5 #32 Evaluate the indefinite integral.

$$\begin{aligned} \int \frac{\sin x}{1 + \cos^2 x} dx &= \int \frac{-1}{1 + u^2} du \\ u = \cos x & \\ du = -\sin x dx & \\ -du = \sin x dx & \\ &= -\arctan u + C \\ &= -\arctan(\cos x) + C \end{aligned}$$

Question 3. (4 marks) §5.5 #41 Evaluate the definite integral.

$$\begin{aligned} \int_0^{\pi} \sec^2(t/4) dt &= \int_0^{\pi/4} \sec^2 u du \\ u = t/4 & \\ du = \frac{1}{4} dt & \\ u(\pi) = \pi/4 & \\ u(0) = 0/4 = 0 & \\ &= 4 \left[\tan u \right]_0^{\pi/4} \\ &= 4 \left[\tan \pi/4 - \tan 0 \right] \\ &= 4 \end{aligned}$$