

Quiz 6

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. (5 marks) §6.2 #6 Evaluate the definite integral.

$$\int_0^{2\pi} \sin^2\left(\frac{1}{3}\theta\right) d\theta$$

Question 2. (5 marks) §6.2 #47 Evaluate the indefinite integral.

$$\int \frac{1}{\sqrt{x^2-16}} dx$$

Q1.
$$\int_0^{2\pi} \sin^2\left(\frac{1}{3}\theta\right) d\theta = \int_0^{2\pi} \left(\frac{1 - \cos\left(\frac{2}{3}\theta\right)}{2}\right) d\theta = \frac{1}{2} \int_0^{2\pi} 1 - \cos\left(\frac{2}{3}\theta\right) d\theta$$

$$= \frac{1}{2} \left[\theta - \frac{\sin\left(\frac{2}{3}\theta\right)}{\frac{2}{3}} \right]_0^{2\pi}$$

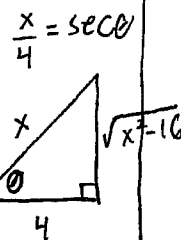
$$= \frac{1}{2} \left[2\pi - \frac{3}{2} \sin\left(\frac{4\pi}{3}\right) \right]$$

$$- \frac{1}{2} \left[0 - \frac{3}{2} \sin 0 \right]$$

$$= \pi + \frac{3}{4} \frac{\sqrt{3}}{2} = \pi + \frac{3\sqrt{3}}{8}$$

Q2.
$$\int \frac{1}{\sqrt{x^2-16}} dx$$

$x = 4\sec\theta$
 $dx = 4\sec\theta \tan\theta d\theta$



$$= \int \frac{1}{\sqrt{(4\sec\theta)^2 - 16}} 4\sec\theta \tan\theta d\theta$$

$$= \int \frac{1}{\sqrt{16\sec^2\theta - 16}} 4\sec\theta \tan\theta d\theta$$

$$= \int \frac{1}{\sqrt{16(\sec^2\theta - 1)}} 4\sec\theta \tan\theta d\theta$$

$$= \int \frac{1}{\sqrt{16\tan^2\theta}} 4\sec\theta \tan\theta d\theta$$

$$= \int \frac{1}{4\tan\theta} 4\sec\theta \tan\theta d\theta$$

$$= \int \sec\theta d\theta$$

$$= \ln|\sec\theta + \tan\theta| + C$$

$$= \ln\left| \frac{x}{4} + \frac{\sqrt{x^2-16}}{4} \right| + C$$