

Quiz 7

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. (5 marks) §6.3 #39 Make a substitution to express the integrand as a rational function and then evaluate the integral.

$$\int \frac{e^{2x}}{e^{2x} + 3e^x + 2} dx$$

Question 2. (5 marks) §6.6 #47 Determine whether the integral is convergent or divergent. Evaluate if it convergent.

$$\int_6^8 \frac{4}{(x-6)^3} dx = \lim_{a \rightarrow 6^+} \int_a^8 \frac{4}{(x-6)^3} dx = \lim_{a \rightarrow 6^+} \int_{a-6}^2 \frac{4}{u^3} du$$

$u = x-6$ $u(8) = 8-6 = 2$
 $du = dx$ $u(a) = a-6$

$$= \lim_{a \rightarrow 6^+} \left[\frac{-2}{u^2} \right]_{a-6}^2$$

$$\int \frac{e^x e^x}{(e^x)^2 + 3e^x + 2} dx \quad \begin{matrix} u = e^x \\ du = e^x dx \end{matrix}$$

\therefore the integral is divergent.

$$= \int \frac{u}{u^2 + 3u + 2} du = \int \frac{2}{u+2} - \frac{1}{u+1} du = 2 \ln|u+2| - \ln|u+1| + C$$

$$= 2 \ln|e^x + 2| - \ln|e^x + 1| + C$$

$$\frac{u}{u^2 + 3u + 2} = \frac{u}{(u+2)(u+1)} = \frac{A_1}{u+2} + \frac{A_2}{u+1}$$

$$\frac{u(u+2)(u+1)}{(u+2)(u+1)} = \frac{A_1(u+1)(u+2)}{(u+2)} + \frac{A_2(u+1)(u+2)}{(u+1)}$$

$$u = A_1(u+1) + A_2(u+2)$$

Let $u = -1$: $-1 = A_1(-1+1) + A_2(-1+2)$
 $-1 = A_2$

Let $u = -2$: $-2 = A_1(-2+1) + A_2(-2+2)$
 $2 = A_1$