

Quiz 7

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. (5 marks) §6.3 #40 Make a substitution to express the integrand as a rational function and then evaluate the integral.

$$\int \frac{\sin x}{\cos^2 x - 3 \cos x} dx$$

Question 2. (5 marks) §6.6 #25 Determine whether the integral is convergent or divergent. Evaluate if it convergent.

$$\int_{-2}^{14} \frac{dx}{\sqrt[4]{x+2}} = \lim_{a \rightarrow -2^+} \int_a^{14} \frac{1}{\sqrt[4]{x+2}} dx = \lim_{a \rightarrow -2^+} \int_{a+2}^{16} \frac{1}{\sqrt[4]{u}} du$$

$$\int \frac{\sin x dx}{\cos^2 x - 3 \cos x}$$

$u = \cos x \quad u(14) = 14 + 2 = 16$
 $du = -\sin x dx \quad u(a) = a + 2$
 $-du = \sin x dx$

$$= \int \frac{-1}{u^2 - 3u} du$$

$$= \int \frac{1}{3u - u^2} du = \int \frac{1/3}{u} + \frac{1/3}{3-u} du$$

$$= \frac{1}{3} \ln|u| - \frac{1}{3} \ln|3-u| + C = \frac{1}{3} \ln|\cos x| - \frac{1}{3} \ln|3 - \cos x| + C$$

$$= \lim_{a \rightarrow -2^+} \left[\frac{4u^{3/4}}{3} \right]_{a+2}^{16}$$

$$= \lim_{a \rightarrow -2^+} \left[\frac{4}{3} 16^{3/4} - (a+2)^{3/4} \right]$$

converges to $\frac{32}{3}$

$$\frac{1}{3u - u^2} = \frac{1}{u(3-u)} = \frac{A_1}{u} + \frac{A_2}{3-u}$$

$$\frac{1 \cdot u(3-u)}{u(3-u)} = \frac{A_1 u(3-u)}{u} + \frac{A_2 u(3-u)}{3-u}$$

$$1 = A_2 u + A_1(3-u)$$

Let $u=0$: $1 = A_2(0) + A_1(3-0)$

$$\frac{1}{3} = A_1$$

Let $u=3$: $1 = A_2(3) + A_1(3-3)$

$$\frac{1}{3} = A_2$$