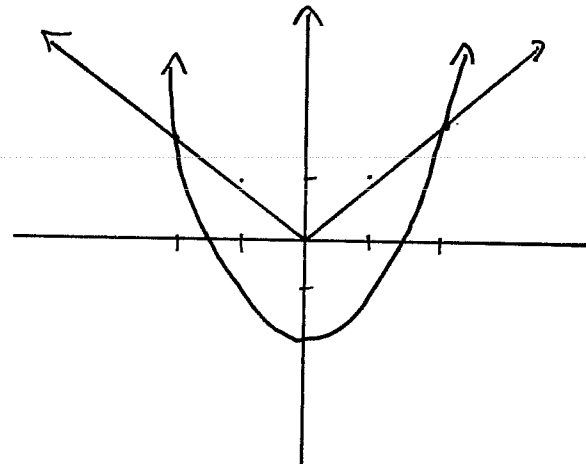


Quiz 8

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. (5 marks) §7.1 #18 Sketch the region enclosed by the given curves and find its area.

$$y = |x|, \quad y = x^2 - 2$$



Question 2. (5 marks) §7.4 #9 Find the exact length of the curve

$$y = \frac{x^3}{3} + \frac{1}{4x}, \quad 1 \leq x \leq 2$$

$$S = \int_a^b \sqrt{1 + (y')^2} dx$$

$$y' = x^2 - \frac{1}{4x^2}$$

$$= \int_1^2 \sqrt{1 + \left(x^2 - \frac{1}{4x^2}\right)^2} dx$$

$$= \int_1^2 \sqrt{1 + x^4 - \frac{1}{2} - \frac{1}{16x^4}} dx$$

$$= \int_1^2 \sqrt{x^4 - \frac{1}{2} - \frac{1}{16x^4}} dx$$

$$= \int_1^2 \sqrt{\left(x^2 - \frac{1}{4x^2}\right)^2} dx$$

$$= \int_1^2 \left|x^2 - \frac{1}{4x^2}\right| dx$$

$$= \int_1^2 \left(x^2 - \frac{1}{4x^2}\right) dx \quad \rightarrow = \frac{2^3}{3} + \frac{1}{4(3)} - \frac{1}{3} - \frac{1}{4}$$

$$= \left[\frac{x^3}{3} + \frac{1}{4x}\right]_1^2 = \frac{13}{6}$$

If $x > 0$ then $y = |x|$ is $y = x$
 and intersection is

$$x = x^2 - 2$$

$$0 = x^2 - x - 2$$

$$0 = (x-2)(x+1)$$

$$x = 2$$

If $x < 0$ then $y = |x|$ is $y = -x$
 and intersection is

$$-x = x^2 - 2$$

$$0 = x^2 + x - 2$$

$$0 = (x+2)(x-1)$$

$$x = -2$$

$$A = \int_{-2}^0 -x - (x^2 - 2) dx + \int_0^2 x - (x^2 - 2) dx$$

$$= \left[-\frac{x^2}{2} - \frac{x^3}{3} + 2x\right]_{-2}^0 + \left[\frac{x^2}{2} - \frac{x^3}{3} + 2x\right]_0^2$$

$$= -\left[-\frac{(-2)^2}{2} - \frac{(-2)^3}{3} + 2(-2)\right]$$

$$+ \left[\frac{2^2}{2} - \frac{2^3}{3} + 2(2)\right] = \frac{20}{3}$$