

## Quiz 8

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

**Question 1.** (5 marks) §7.1 #19 Sketch the region enclosed by the given curves and find its area.

$$y = 1/x, y = x, y = \frac{1}{4}x, x > 0$$

**Question 2.** (5 marks) §7.4 #10 Find the exact length of the curve

$$x = \frac{y^4}{8} + \frac{1}{4y^2}, 1 \leq y \leq 2$$

$$S = \int_a^b \sqrt{1 + (x')^2} dy$$

$$x' = \frac{y^3}{2} - \frac{1}{2y^3}$$

$$= \int_1^2 \sqrt{1 + \left(\frac{y^3}{2} - \frac{1}{2y^3}\right)^2} dy$$

$$= \int_1^2 \sqrt{1 + \frac{y^6}{4} - \frac{1}{2} + \frac{1}{4y^6}} dy$$

$$= \int_1^2 \sqrt{\frac{y^6}{4} + \frac{1}{2} + \frac{1}{4y^6}} dy$$

$$= \int_1^2 \sqrt{\left(\frac{y^3}{2} + \frac{1}{2y^3}\right)^2} dy$$

$$= \int_1^2 \left|\left(\frac{y^3}{2} + \frac{1}{2y^3}\right)\right| dy$$

$$= \int_1^2 \frac{y^3}{2} + \frac{1}{2y^3} dy$$

$$= \left[ \frac{y^4}{8} - \frac{1}{4y^2} \right]_1^2$$

Intersection of  
 $y = x$  and  $y = \frac{1}{x}$

$$x = \frac{1}{x}$$

$$x^2 = 1$$

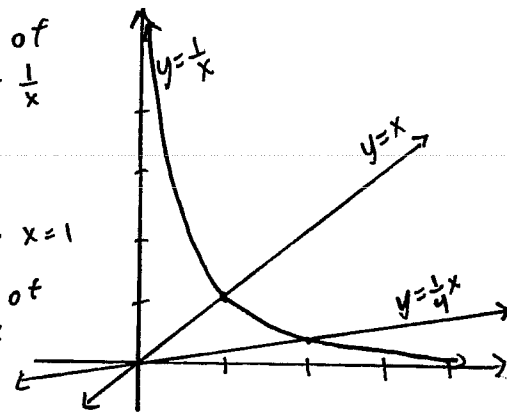
$$x = \pm 1 \text{ so } x = 1$$

Intersection of  
 $y = \frac{1}{x}$  and  $y = \frac{1}{4}x$

$$\frac{1}{x} = \frac{1}{4}x$$

$$4 = x^2$$

$$\pm 2 = x \text{ so } x = 2$$



$$A = \int_0^1 x - \frac{1}{4}x dx + \int_1^2 \frac{1}{x} - \frac{1}{4}x dx$$

$$= \left[ \frac{x^2}{2} - \frac{1}{8}x^2 \right]_0^1 + \left[ \ln|x| - \frac{1}{8}x^2 \right]_1^2$$

$$= \frac{1}{2} - \frac{1}{8} + \ln 2 - \frac{1}{8}2^2 - \left[ \ln 1 - \frac{1}{8}(1)^2 \right]$$

$$= \ln 2$$

$$= \frac{2^4}{8} - \frac{1}{4 \cdot 2^2} - \left[ \frac{1}{8} - \frac{1}{4} \right]$$

$$= 2 - \frac{1}{16} - \frac{1}{8} + \frac{1}{4} = \frac{33}{16}$$