Name: ------------------

Student ID: ------------------

Test 1

This test is graded out of 45 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Formulae:

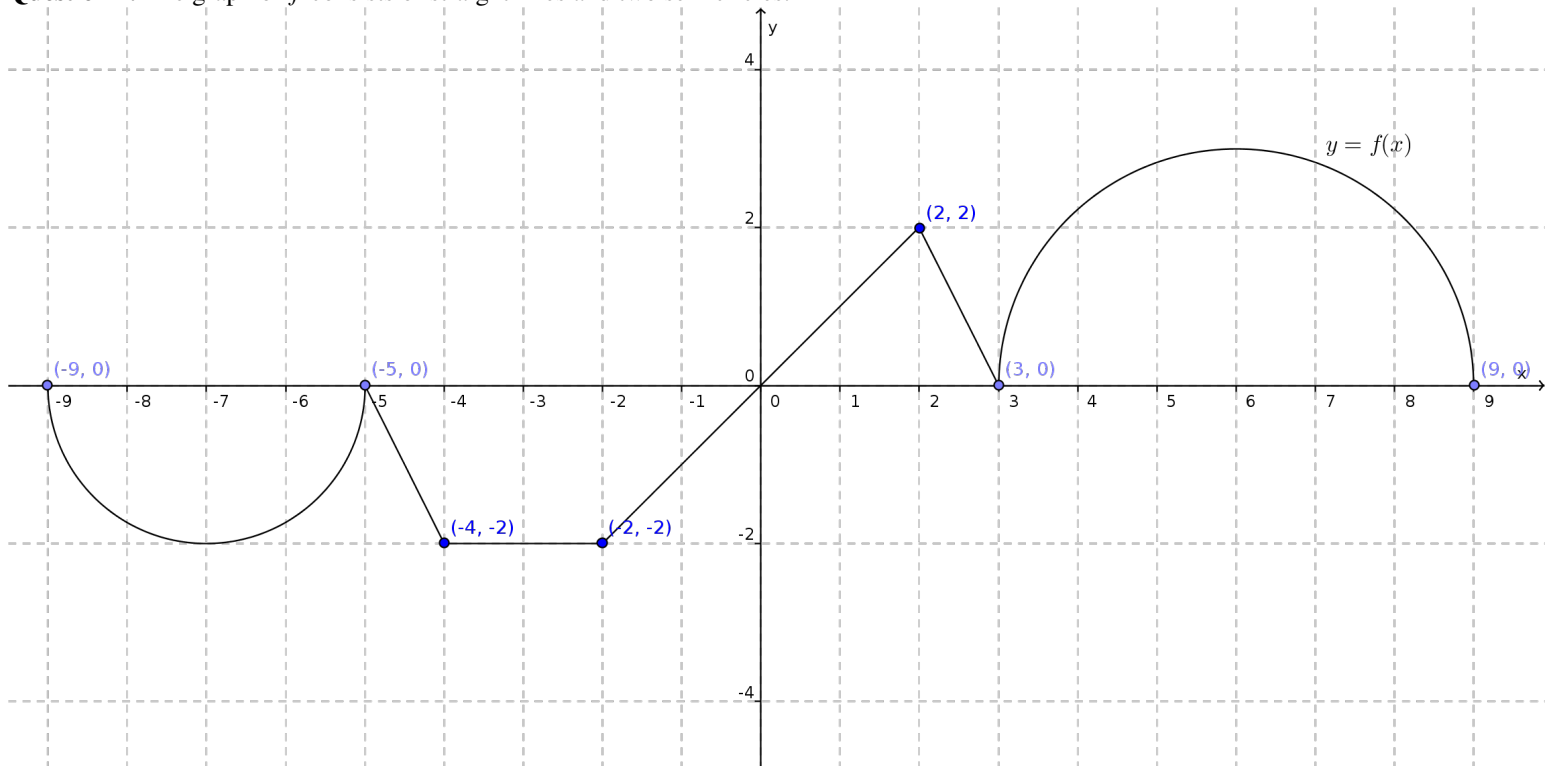$$\sum_{i=1}^{n} c = cn \text{ where } c \text{ is a constant} \qquad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6} \qquad \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

Question 1. *(5 marks)* Evaluate the definite integral of $f(x) = -3x^2 + 1$ on $[-1, 3]$ using the definition of the definite integral.

Question 2. The graph of f consists of straight lines and two semicircles.

Use the graph of the find the exact value of the following integrals.

a. *(1 mark)*

$$\int_{-9}^{-7} f(x)\, dx$$

b. *(1 mark)*

$$\int_{-1}^{1} f(x)\, dx$$

c. *(1 mark)*

$$\int_{-3}^{0} f(x)\, dx$$

d. *(1 mark)*

$$\int_{0}^{3} f(x)\, dx$$

e. *(1 mark)*

$$\int_{-3}^{3} f(x)\, dx$$

Question 3. *(5 marks)* If

$$\int_3^5 (f(x) - 2)\, dx = 7, \quad \int_3^0 2f(x)\, dx = 3,$$

and $f(x) = f(-x)$ for all $x \in \mathbb{R}$ find

$$\int_{-5}^5 f(x)\, dx.$$

Question 4. *(5 marks)* Evaluate the indefinite integral:

$$\int \frac{x^5}{\sqrt{x^3 + \pi}}\, dx$$

Question 5. Given

$$h(x) = \int_{\arctan 3x}^{\cot 2x} u\sqrt[3]{\sin u}\, du$$

a. *(2 marks)* Rewrite $h(x)$ as the sum of two integrals with a constant as the lower bound.

b. *(1 mark)* Rewrite the two integrals of part a. as composite functions with an integral as the outer function.

c. *(2 marks)* Using part b. and the 2$^{\text{nd}}$ FTC determine $h'(x)$.

Question 6. Given

$$f(x) = e^{3x}, \quad [0, \ln 2]$$

a. *(2 marks)* Find the average value of f on the given interval.

b. *(2 marks)* Find c such that $f_{ave} = f(c)$.

c. *(1 mark)* Sketch the graph of f and a rectangle whose area is the same as the area under the graph of f.

Question 7. *(5 marks)* Estimate the definite integral of $f(x) = \sin x$ from $x = 0$ to $x = 2\pi/3$ using two rectangles and using the midpoints. Sketch the curve and the approximating rectangles.

Question 8. *(5 marks)* If f is continuous on $\mathbb{R}$, prove that

$$\int_a^b f(-x)\, dx = \int_{-b}^{-a} f(x)\, dx$$

For the case where $f(x) \geq 0$ and $0 < a < b$, draw a diagram to interpret this equation geometrically as an equality of areas.

Question 9. *(5 marks)* Evaluate the definite integral

$$\int_{-5}^{1} \ln(4 - 3x) \, dx$$

Bonus Question. *(3 marks)*
The Fresnel function is defined as

$$S(x) = \int_{0}^{x} \sin\left(\frac{\pi t^2}{2}\right) dt.$$

At what values of x does this function have local maximum values, Justify.