Name: _____
Student ID: _____

Test 1

This test is graded out of 45 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

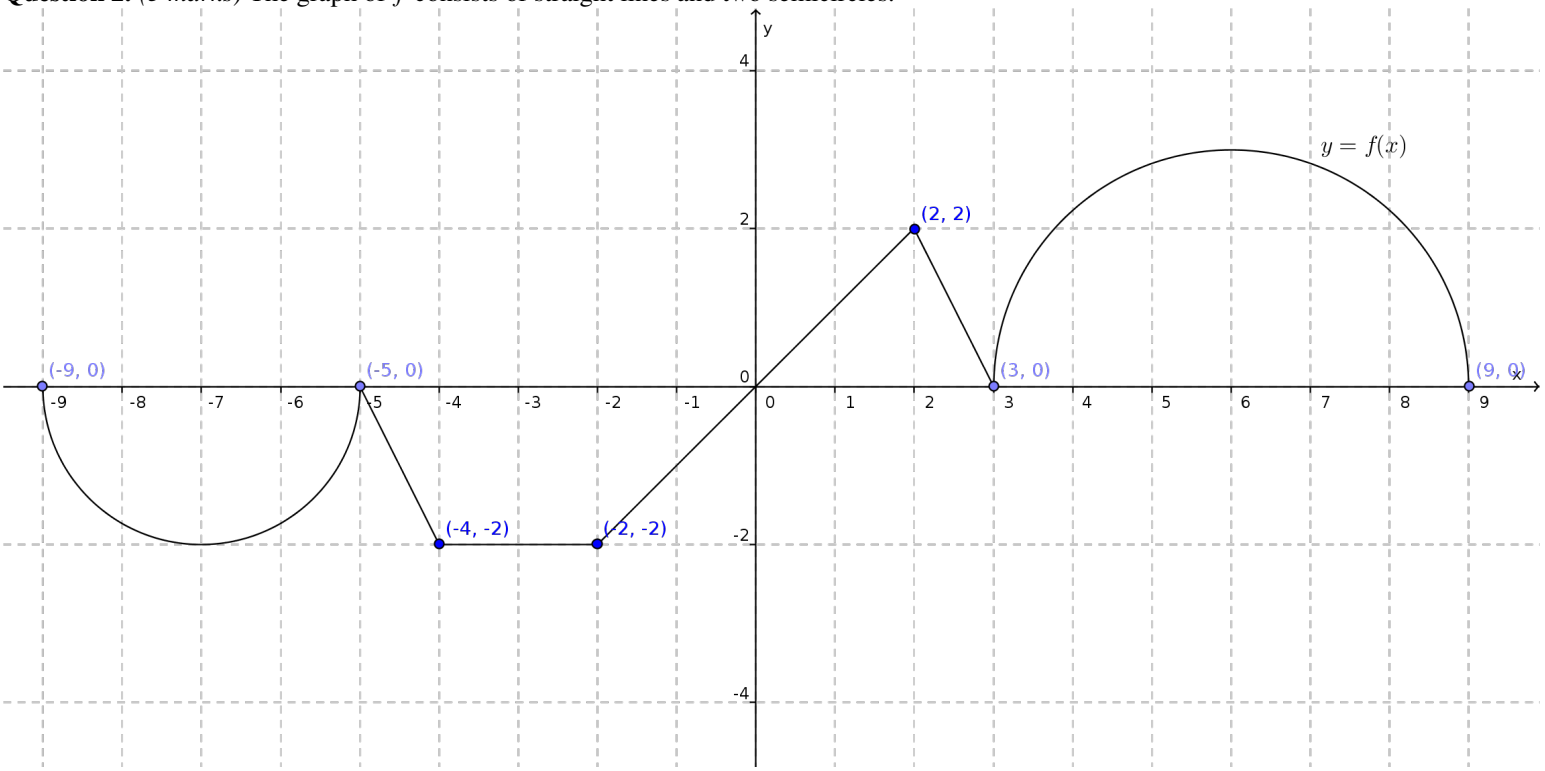
Formulae:

$\sum_{i=1}^{n} c = cn$ where c is a constant $\quad \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$

$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$ $\quad \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$

Question 1. *(5 marks)* Evaluate the definite integral of $f(x) = -6x^2 + 3$ on $[-1, 1]$ using the definition of the definite integral.

Question 2. *(5 marks)* The graph of f consists of straight lines and two semicircles.



Use the graph of the find the exact value of the following integrals.

a. *(1 mark)*

$$\int_2^6 f(x)\,dx$$

b. *(1 mark)*

$$\int_{-2}^2 f(x)\,dx$$

c. *(1 mark)*

$$\int_{-5}^{-3} f(x)\,dx$$

d. *(1 mark)*

$$\int_{-2}^6 f(x)\,dx$$

e. *(1 mark)*

$$\int_{-7}^{-5} f(x)\,dx$$

Question 3. *(5 marks)* Evaluate the indefinite integral:

$$\int x^5 \sqrt{x^3 + e}\, dx$$

Question 4. Given

$$h(x) = \int_{\text{arcsec}2x}^{\tan 3x} u \sqrt[3]{\sin u}\, du$$

a. *(2 marks)* Rewrite $h(x)$ as the sum of two integrals with a constant as the lower bound.

b. *(1 mark)* Rewrite the two integrals of part a. as composite functions with an integral as the outer function.

c. *(2 marks)* Using part b. and the 2nd FTC determine $h'(x)$.

Question 5. Given

$$f(x) = e^{2x}, \quad [0, \ln 3]$$

a. *(2 marks)* Find the average value of f on the given interval.

b. *(2 marks)* Find c such that $f_{ave} = f(c)$.

c. *(1 mark)* Sketch the graph of f and a rectangle whose area is the same as the area under the graph of f.

Question 6. *(5 marks)* Estimate the definite integral of $f(x) = \cos x$ from $x = -\pi/3$ to $x = \pi/3$ using two rectangles and using the midpoints. Sketch the curve and the approximating rectangles.

Question 7. *(5 marks)* If f is continuous on $\mathbb{R}$, prove that

$$\int_a^b f(x+c)\,dx = \int_{a+c}^{b+c} f(x)\,dx.$$

For the case where $f(x) \geq 0$, draw a diagram to interpret this equation geometrically as an equality of areas.

Question 8. *(5 marks)* Evaluate the definite integral

$$\int_{-3}^{1} \ln(3-2x)\,dx$$

Question 9. *(5 marks)* If

$$\int_2^7 (f(x)-3)\,dx = 5, \quad \int_2^0 3f(x)\,dx = 2,$$

and $f(x) = f(-x)$ for all $x \in \mathbb{R}$ find

$$\int_{-7}^7 f(x)\,dx.$$

Bonus Question. *(3 marks)*
The Fresnel function is defined as

$$S(x) = \int_0^x \sin\left(\frac{\pi t^2}{2}\right).$$

At what values of x does this function have local minimum values, Justify.