

Name: \_\_\_\_\_  
Student ID: \_\_\_\_\_

## Test 2

This test is graded out of 45 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

**Question 1.** (5 marks) Evaluate the definite integral

$$\int_{\ln \sqrt[3]{\pi/12}}^{\ln \sqrt[3]{\pi/2}} e^{3x} \cos^2 e^{3x} dx$$

**Question 2.** (5 marks) Evaluate the indefinite integral

$$\int \frac{x^3 + x^2 + 1}{x^3 + x^2 + 2x} dx$$

**Question 3.** (5 marks) Find the exact length of the curve

$$y = \ln(\cos x), \quad 0 \leq x \leq \frac{\pi}{3}$$

**Question 4.** (5 marks) Sketch the region(s) enclosed by the given curves and set up the integral to find the total area of the region(s).

$$y = |x|, y = \sqrt{x}, y = -(x + 1)^2 + 1$$

**Question 5.** (5 marks) Determine whether the integral is convergent or divergent. Evaluate the integral if convergent.

$$\int_{\sqrt{3}}^{2\sqrt{3}} \frac{1}{x\sqrt{x^2-3}} dx$$

**Question 6.** (5 marks) Evaluate the definite integral

$$\int_3^5 \sqrt{x^2 - 2x - 3} \, dx$$

given that

$$\int_0^{\pi/3} \sec^3 x \, dx = \sqrt{3} + \frac{1}{2} \ln(2 + \sqrt{3}).$$

**Question 7.** (5 marks) Set up the integral to find the volume of the solid obtained from the region bounded by the graphs of  $y = (e - 1)\sqrt{x} + 1$ ,  $y = e^x$  rotated about the line  $x = -1$ . Sketch the region, draw a representative rectangle, write a representative element and the integral.

**Question 8.** (5 marks) Find the value of the constant  $C$  for which the integral

$$\int_0^{\infty} \left( \frac{6x}{x^2+3} - \frac{C}{3x+2} \right) dx$$

converges. Evaluate the integral for this value of  $C$ .



**Question 9.** (5 marks) Evaluate the indefinite integral

$$\int \frac{dx}{x\sqrt{5-x^2}}$$

**Bonus Question.** (3 marks) Find the exact length of the curve

$$y = \int_2^{\frac{e^x+1}{e^x-1}} \frac{1}{z} dz \quad [\ln 2, \ln 3]$$