Name: -------------------

Student ID: -------------------

Test 2

This test is graded out of 45 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(5 marks)* Evaluate the definite integral

$$\int_{5}^{\frac{6}{\sqrt{3}}+2} \sqrt{x^2 - 4x - 5} \, dx$$

given that

$$\int_{0}^{\pi/6} \sec^3 x \, dx = \frac{1}{3} + \frac{\ln 3}{4}.$$

Question 2. *(5 marks)* Determine whether the integral is convergent or divergent. Evaluate the integral if convergent.

$$\int_{\sqrt{5}}^{2\sqrt{5}} \frac{1}{x\sqrt{x^2 - 5}}\, dx$$

Question 3. *(5 marks)* Set up the integral to find the volume of the solid obtained from the region bounded by the graphs of $y = (e-1)\sqrt{x}+1$, $y = e^x$ rotated about the line $x = 2$. Sketch the region, draw a representative rectangle, write a representative element and the integral.

Question 4. *(5 marks)* Find the value of the constant C for which the integral

$$\int_0^\infty \left(\frac{8x}{x^2+4} - \frac{C}{4x+3} \right) dx$$

converges. Evaluate the integral for this value of C.

Question 5.*(5 marks)* Evaluate the indefinite integral

$$\int \frac{x^3 + x^2 + 2}{x^3 + x^2 + x} \, dx$$

Question 6.*(5 marks)* Evaluate the definite integral

$$\int_{\ln \sqrt[5]{\pi/6}}^{\ln \sqrt[5]{\pi/2}} e^{5x} \sin^2 e^{5x} \, dx$$

Question 7. *(5 marks)* Sketch the region(s) enclosed by the given curves and set up the integral to find the total area of the region(s).

$$y = -|x+1| + 1, \ y = \sqrt{x}, \ y = x^2$$

Question 8. *(5 marks)* Evaluate the indefinite integral

$$\int \frac{dx}{x\sqrt{2-x^2}}$$

Question 9. *(5 marks) (5 marks)* Find the exact length of the curve

$$y = \ln(\sec x), \quad 0 \leq x \leq \frac{\pi}{4}$$

Bonus Question. *(3 marks)*

Find the exact length of the curve

$$y = \int_2^{\frac{e^x+1}{e^x-1}} \frac{1}{z} \, dz \quad [\ln 2, \ln 3]$$