Dawson College: Linear Algebra (SCIENCE): 201-NYA-05-S4: Winter 2015

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Quiz 2

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §23.2 #13 *(5 marks)* Use the delta method to find a general expression for the slope of a tangent line to each of the indicated curves. Then find the slopes for the given values of x. Sketch the curves and tangent lines.

$y = 6x - x^2$; $x = -2$, $x = 3$

$$y' = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h\to 0} \frac{6(x+h) - (x+h)^2 - (6x - x^2)}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{6x} + 6h - \cancel{x^2} - 2xh - h^2 - \cancel{6x} + \cancel{x^2}}{h}$$

$$= \lim_{h\to 0} \frac{\cancel{h}(6 - 2x - h)}{\cancel{h}} = 6 - 2x$$

@ $x = -2$

$y'(-2) = 6 - 2(-2) = 10 = m_{tan}$

$y(-2) = 6(-2) - (-2)^2 = -16$

$\therefore (-2, -16)$ on tangent and curve

$y = mx + b$

$y = 10x + b$

$-16 = 10(-2) + b$

$b = 4$

$\therefore y = 10x + 4$ is the tangent to the curve at $x = -2$

@ $x = 3$

$y'(3) = 6 - 2(3) = 0$

$y(3) = 6(3) - 3^2 = 9$

$\therefore (3, 9)$ on tangent and curve

$y = mx + b$ $\therefore y = 9$ is the tangent to the curve at $x = 3$

x-int:

$0 = 6x - x^2$

$0 = x(6 - x)$

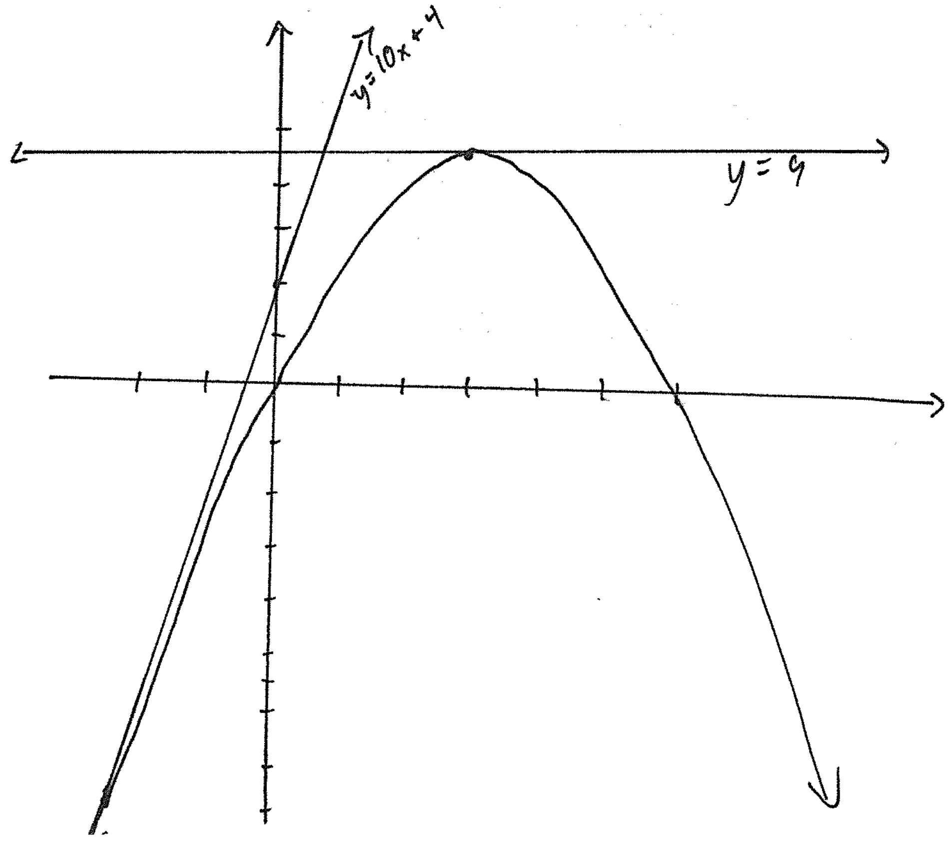
$x = 0$ $x = 6$

y-int: $y = 6(0) - 0^2 = 0$

vertex: $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$

$= (3, f(3))$

$= (3, 9)$



Question 2. §23.3 #11 *(2 marks)* Find the derivative of each of the functions by using the definition.

$f(x) = y = x^2 - 7x$

$$y' = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^2 - 7(x+h) - [x^2 - 7x]}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^2} + 2xh + h^2 - \cancel{7x} - 7h - \cancel{x^2} + \cancel{7x}}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{h}[2x + h - 7]}{\cancel{h}}$$

$$= 2x - 7$$

Question 3. §23.3 #20 *(3 marks)* Find the derivative of each of the functions by using the definition.

$f(x) = y = \frac{x}{x-1}$

$$y' = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \lim_{h \to 0} \frac{\frac{x+h}{x+h-1} - \frac{x}{x-1}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{(x+h)(x-1)}{(x+h-1)(x-1)} - \frac{x(x+h-1)}{(x+h-1)(x-1)}}{h}$$

$$= \lim_{h \to 0} \frac{\frac{\cancel{x^2} - \cancel{x} + \cancel{xh} - h - \cancel{x^2} - \cancel{xh} + \cancel{x}}{(x+h-1)(x-1)}}{h}$$

$$= \lim_{h \to 0} \frac{-\cancel{h}}{(x+h-1)(x-1)\cancel{h}}$$

$$= \frac{-1}{(x-1)^2}$$