

Quiz 4

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §23.7 #31 (3 marks) Find the derivative.

$$\begin{aligned}
 f(R) &= \sqrt{\frac{2R+1}{4R+1}} = \left(\frac{2R+1}{4R+1}\right)^{1/2} \\
 f'(R) &= \frac{1}{2} \left(\frac{2R+1}{4R+1}\right)^{-1/2} \frac{(2)(4R+1) - (2R+1)(4)}{(4R+1)^2} \\
 &= \frac{1}{2} \sqrt{\frac{4R+1}{2R+1}} \frac{8R+2 - 8R-4}{(4R+1)^2} \\
 &= \frac{1}{2} \sqrt{\frac{4R+1}{2R+1}} \frac{-2}{(4R+1)^2} = \frac{-1}{\sqrt{2R+1} (4R+1)^{3/2}}
 \end{aligned}$$

Question 2. §23.8 #19 (4 marks) At what point(s) does the graph of $x^2 + y^2 = 4x$ have a horizontal tangent?

Horizontal tangent when $m_{\text{tan}} = 0$. $m_{\text{tan}} = y' = 0$

$$\frac{d}{dx} [x^2 + y^2] = \frac{d}{dx} [4x] \qquad \frac{2-x}{y} = 0$$

$$2x + 2yy' = 4$$

$$2yy' = 4 - 2x$$

$$y' = \frac{4-2x}{2y} = \frac{2-x}{y}$$

$$2-x = 0$$

$$2 = x$$

sub into equation to find points

$$2^2 + y^2 = 4(2)$$

$$y^2 = 4$$

$$y = \pm 2$$

∴ two points at (2, 2) and (2, -2)

Question 3. §23.9 #29 (3 marks) Find the second derivative.

$$x^2 - xy = 1 - y^2$$

$$\frac{d}{dx} [x^2 - xy] = \frac{d}{dx} [1 - y^2]$$

$$2x - y - xy' = -2yy'$$

$$2x - y = xy' - 2yy'$$

$$2x - y = y'(x - 2y)$$

$$y' = \frac{2x - y}{x - 2y}$$

$$\frac{d}{dx} [y'] = \frac{d}{dx} \left[\frac{2x - y}{x - 2y} \right]$$

$$\begin{aligned}
 y'' &= \frac{(2 - y')(x - 2y) - (2x - y)(-2y')}{(x - 2y)^2} \\
 &= \frac{2x - 4y - xy' + 2yy' - [2x - 4xy' - y + 2yy']}{(x - 2y)^2} \\
 &= \frac{-3y + 3xy'}{(x - 2y)^2}
 \end{aligned}$$

$$= \frac{-3y + 3x \left(\frac{2x-y}{x-2y} \right)}{(x-2y)^2}$$

$$= \frac{-3y(x-2y) + 3x(2x-y)}{(x-2y)^3}$$

$$= \frac{-3xy + 6y^2 + 6x^2 - 3xy}{(x-2y)^3}$$

$$= \frac{6y^2 - 6xy + 6x^2}{(x-2y)^3}$$