

Quiz 6

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §24.4 #31 (5 marks) One statement of Boyle's law is that the pressure of a gas varies inversely as the volume for constant temperature. If a certain gas occupies 650 cm³ when the pressure is 230 kPa and the volume is increasing at the rate of 20.0 cm³/min, how fast is the pressure changing when the volume is 810 cm³?

① P - pressure $P_1 = 230 \text{ kPa}$
 V - volume $V_1 = 650 \text{ cm}^3$
 $V_2 = 810 \text{ cm}^3$

$$\frac{dP}{dt} = \frac{-K}{V^2} \cdot \frac{dV}{dt}$$

where $K = P_1 V_1$
 $= (230)(650)$

So $\frac{dP}{dt} = \frac{-(230)(650)}{(810)^2} \cdot 20.0$

$= 4.6 \text{ kPa/min}$

$\frac{dP_2}{dt} = ?$

③ $P \propto \frac{1}{V}$

$P = \frac{K}{V}$

④ $\frac{d}{dt}[P] = \frac{d}{dt}\left[\frac{K}{V}\right]$

Question 2. §24.5 #31 (5 marks) Sketch the graphs of the given functions by determining the appropriate information and points from the first and second derivatives.

$y = x^5 - 5x$

x-int: $0 = y$

$0 = x^5 - 5x$

$0 = x(x^4 - 5)$

$x = 0$ $x = \pm \sqrt[4]{5} \approx \pm 1.5$

first derivative:

$y' = 5x^4 - 5$

critical points:

$0 = y'$

$0 = 5x^4 - 5$

$0 = x^4 - 1$

$0 = (x^2 - 1)(x^2 + 1)$

$0 = (x-1)(x+1)(x^2+1)$

$x = 1$ $x = -1$

intervals	$(-\infty, -1)$	$(-1, 1)$	$(1, \infty)$
test point, p	-2	0	2
$y'(p)$	75	-5	75
+/-	+	-	+
inc/dec	↗	↘	↗

∴ rel. max @ $x = -1$ $y = (-1)^5 - 5(-1) = 4$



∴ rel. min @ $x = 1$ $y = 1^5 - 5(1) = -4$

Second derivative:

$y'' = 20x^3$

values of x when $y'' = 0$:

$0 = 20x^3$
 $0 = x$

intervals	$(-\infty, 0)$	$(0, \infty)$
test point, p	-1	1
$y''(-1)$	-20	20
+/-	-	+
concavity		

∴ inflection point at $x=0$ $y=0$

