Dawson College: Calculus (CHEM. TECH.): 201-NYA-05-S4: Winter 2015

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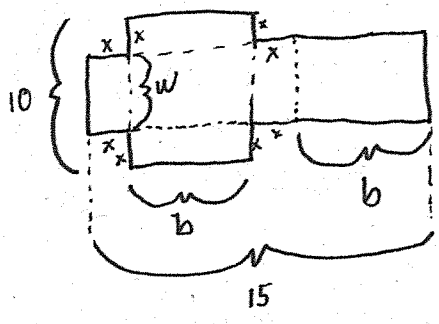
Quiz 7

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §24.4 #33 *(4 marks)* A box with a lid is to be made from a rectangular piece of card board 10*cm* by 15*cm*, as shown on the board. Two equal squares of side x are to be removed from one end, and two equal rectangles are to be removed from the other end so that the tabs can be folded to form the box with a lid. Find x such that the volume of the box is a maximum.



$V = x(w)(b) = xwb$ (to maximize)

$10 = w + 2x \Leftrightarrow w = 10 - 2x$

$15 = 2b + 2x \Leftrightarrow b = \frac{15-2x}{2}$

So $V = x(10-2x)\left(\frac{15-2x}{2}\right) = x(5-x)(15-2x)$

$= x(2x^2 - 15x - 10x + 75)$

$= 2x^3 - 25x^2 + 75x$

Find the critical points

$V' = 6x^2 - 50x + 75$

$0 = 6x^2 - 50x + 75$

$x = \frac{50 \pm \sqrt{(50)^2 - 4(6)(75)}}{2(6)}$

$= \frac{50 \pm \sqrt{700}}{12} = \frac{50 \pm 10\sqrt{7}}{12} \doteq 6.37$ or 1.96

Lets use the second derivative test to determine which critical point is a max.

$V'' = 12x - 50$

$V''(6.37) = 12(6.37) - 50 = 26.44 > 0$ min

$V''(1.96) = 12(1.96) - 50 = -26.48 < 0$ max

∴ Volume is maximize when $x = 1.96$.

Question 2. §24.8 #19 *(3 marks)* Find the values of Δy and dy for the given values of x and dx.

$y = x\sqrt{1+4x}, x = 12, \Delta x = 0.06$

$y' = \sqrt{1+4x} + \frac{x}{\sqrt{1+4x}} \cdot 4 \cdot \frac{1}{2}$

So $y'(12) = \sqrt{1+4(12)} + \frac{12(2)}{\sqrt{1+4(12)}}$

$= 7 + \frac{24}{7}$

$= \frac{49+24}{7} = \frac{73}{7}$

$dy = y'dx = y'\Delta x = \frac{73}{7}(0.06) \doteq 0.6257$

$\Delta y = f(x+\Delta x) - f(x)$

$= f(12.06) - f(12)$

$= (12.06)\sqrt{1+4(12.06)} - 12\sqrt{1+4(12)}$

$\doteq 0.6265$

Question 3. §27.1 #27 *(3 marks)* Find the derivatives of the given functions.

$y = 2\sin^2 3x \cos 2x$

$y' = 4\sin(3x)\cos(3x)(3)\cos(2x) + 2\sin^2(3x)(-\sin 2x)\cdot 2$

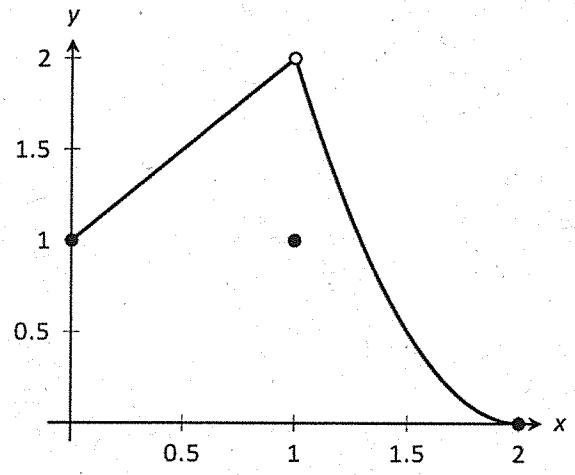
$= 12\sin(3x)\cos(3x)\cos(2x) - 4\sin^2(3x)\sin(2x)$

$= 4(\sin(3x))\left[3\cos(3x)\cos(2x) - \sin(3x)\sin(2x)\right]$

Question 4.

a. *(2 marks)* State the conditions for a function, $f(x)$, to be continuous at $x = a$.

b. *(2 marks)* Determine the values of x for which the function, as represented below by the graph[1] is not continuous. Justify.



see test #1