**Dawson College: Calculus I** (CHEM. TECH.): 201-NYA-05-S04: Winter 2015

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# Test 2

This test is graded out of 49 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1:

Compute the derivative of the following functions *(1 mark each)*:

a. $f(x) = \sin x$ $\quad f'(x) = \cos x$

b. $f(x) = \cos x$ $\quad f'(x) = -\sin x$

c. $f(x) = \csc x$ $\quad f'(x) = -\csc x \cot x$

d. $f(x) = \sec x$ $\quad f'(x) = \sec x \tan x$

e. $f(x) = \tan x$ $\quad f'(x) = \sec^2 x$

f. $f(x) = \cot x$ $\quad f'(x) = -\csc^2 x$

g. $f(x) = \arcsin x$ $\quad f'(x) = \dfrac{1}{\sqrt{1-x^2}}$

h. $f(x) = \arccos x$ $\quad f'(x) = \dfrac{-1}{\sqrt{1-x^2}}$

i. $f(x) = \arctan x$ $\quad f'(x) = \dfrac{1}{x^2+1}$

j. $f(x) = \ln x$ $\quad f'(x) = \dfrac{1}{x}$

k. $f(x) = e^x$ $\quad f'(x) = e^x$

l. $f(x) = \operatorname{arcsec} x$ $\quad f'(x) = \dfrac{1}{x\sqrt{x^2-1}}$

**Question 2.** Let $f(x)=x^2(x+2)^2$

a. *(2 marks)* Determine the $x$ and $y$ intercepts of $f(x)$.
b. *(2 marks)* Determine $f'(x)$ and solve for the critical points.
c. *(2 marks)* State the intervals where $f(x)$ is increasing/decreasing?
d. *(1 mark)* Identify the relative minimum and maximum.
e. *(1 mark)* Find $f''(x)$.
f. *(2 marks)* State the intervals where $f(x)$ is concave up/down?
g. *(1 mark)* Identify any inflection points.
h. *(2 marks)* Clearly sketch the graph of $f(x)$.

a) x-int:

$0=f(x)$

$0=x^2(x+2)^2$

$x=0 \quad x=-2$

y-int:

$(0,f(0))=(0,0)$

b) $f'(x)=2x(x+2)^2+2x^2(x+2)$

$=x(x+2)[2(x+2)+2x]$

$=x(x+2)(4x+4)$

critical points:

$0=f'(x)$

$0=x(x+2)(4x+4)$

$x=0 \quad x=-2 \quad x=-1$

c)

| | $(-\infty,-2)$ | $(-2,-1)$ | $(-1,0)$ | $(0,\infty)$ |
|---|---|---|---|---|
| test point, p | $-3$ | $-3/2$ | $-1/2$ | $1$ |
| $f'(p)$ | $-15$ | | | $21$ |
| +/− | − | + | − | + |
| inc/dec | ↘ | ↗ | ↘ | ↗ |

d) @ $x=-2$ $\quad y=f(-2)=(-2)^2(-2+2)^2=0$ minimum

@ $x=-1$ $\quad y=f(-1)=(-1)^2(-1+2)^2=1$ maximum

@ $x=0$ $\quad y=f(0)=0^2(0+2)^2=0$ minimum

e) $f''(x)=(x+2)(4x+4)+x(4x+4)+4x(x+2)$

$=4x^2+8x+4x+8+4x^2+4x+4x^2+8x$

$=12x^2+24x+8$

f) $0=12x^2+24x+8$

$0=3x^2+6x+2$

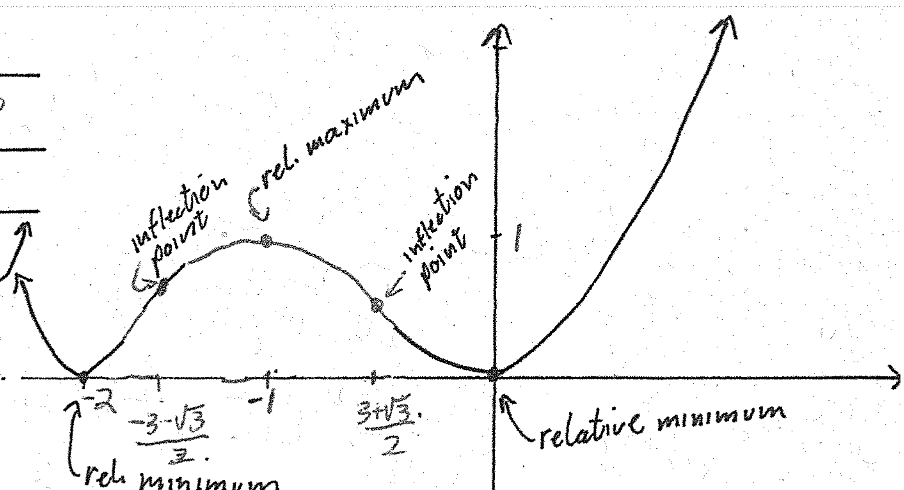
$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}=\dfrac{-6\pm\sqrt{(6)^2-4(3)(2)}}{2(3)}$

$=\dfrac{-6\pm\sqrt{12}}{6}=\dfrac{-3\pm\sqrt{3}}{2}$

| | $\left(-\infty,\frac{-3-\sqrt3}{2}\right)$ | $\left(\frac{-3-\sqrt3}{2},\frac{-3+\sqrt3}{2}\right)$ | $\left(\frac{-3+\sqrt3}{2},\infty\right)$ |
|---|---|---|---|
| test point, p | $-2$ | $-1$ | $0$ |
| $f''(p)$ | $8$ | $-4$ | $8$ |
| +/− | + | − | + |
| concavity | ∪ | ∩ | ∪ |

g) Inflection points at

$x=\dfrac{-3\pm\sqrt{3}}{2}$



**Question 2.** *(5 marks)* Compute the derivative of the following functions. *(Do not simplify.)*

a. *(3 marks)*

$$f(x) = (\arcsin(2x))(\ln(\sec x))$$

b. *(3 marks)*

$$g(x) = \frac{x^2 + \sin x}{e^{3x}}$$

c. *(3 marks)*

$$s(x) = \sqrt{\cot(\arctan(5x))}$$

a) $f'(x) = \frac{1}{\sqrt{1-(2x)^2}} \cdot 2(\ln(\sec x)) + \arcsin(2x) \frac{1}{\sec x} \sec x \tan x$

b) $g'(x) = \frac{(2x + \cos x)e^{3x} - (x^2 + \sin x)e^{3x}(3)}{(e^{3x})^2}$

c) $s'(x) = \frac{1}{2} \cdot \frac{1}{\sqrt{\cot(\arctan(5x))}} (-\csc^2(\arctan(5x))) \cdot \frac{1}{1+(5x)^2} \cdot 5$

**Question 3.** *(5 marks)* One statement of Boyle's law is that the pressure of a gas varies inversely as the volume for constant temperature. If a Yanngon gas occupies 278 $cm^3$ when the pressure is 123 kPa and the volume is increasing at the rate of 5.0 $cm^3$/min, how fast is the pressure changing when the volume is 314 $cm^3$?

① P - pressure
V - volume

$P_1 = 123$ kPa
$V_1 = 278$ $cm^3$
$V_2 = 314$ $cm^3$

$\frac{dV}{dt} = 5.0$ $cm^3$/min

$\frac{dP}{dt} = ?$

③ $P \propto \frac{1}{V}$

$P = \frac{K}{V}$

④ $\frac{d}{dt}[P] = \frac{d}{dt}\left[\frac{K}{V}\right]$

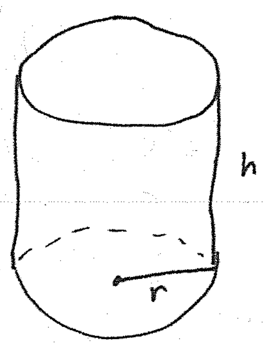
$\frac{dP}{dt} = \frac{-K}{V^2} \cdot \frac{dV}{dt}$

where $K = P_1 V_1 = 123(278)$

So $\frac{dP}{dt} = \frac{-123(278)}{(314)^2} \cdot 5$

$= -1.73$ KPa/min

**Question 5.**[1] *(5 marks)* A standard soda can is roughly cylindrical and holds 355 $cm^3$ of liquid. What dimensions should the cylinder be to minimize the material needed to produce the can?




① $V = \pi r^2 h$ $\quad 355 = \pi r^2 h \Leftrightarrow h = \frac{355}{\pi r^2}$

② $SA = 2\pi r h + 2\pi r^2$

Minimize material so minimize SA.

Sub ① into ②

$SA = 2\cancel{\pi} r \frac{355}{\cancel{\pi} r^{\cancel{2}}} + 2\pi r^2$

$SA = \frac{710}{r} + 2\pi r^2$

Lets find the critical points

$SA' = \frac{-710}{r^2} + 4\pi r$

$0 = SA'$

$0 = \frac{-710}{r^2} + 4\pi r$

$0 = -710 + 4\pi r^3$

$r = \sqrt[3]{\frac{710}{4\pi}} \doteq 3.84$

and $r = 0$

Lets verify if SA is minimized by $r \doteq 3.84$ by the second derivative test.

$SA'' = \frac{1420}{r^3} + 4\pi$

$SA''(3.84) = \frac{1420}{(3.84)^2} + 4\pi$

$\doteq 108.87 > 0$

$\therefore r \doteq 3.84$
$h = 7.66$
minimize the material

[1] from APEX Calculus

**Question 6.** *(3 marks)*

a. *(3 marks)* Find the values of $\Delta y$ and $dy$ for the given values of $x$ and $dx$.

$$y = \ln\left[\sqrt{x+2}(3x+1)^3\right],\ x=2,\ \Delta x = 0.009$$

b. *(2 marks)* Explain the difference between $\Delta y$ and $dy$.

b) $\Delta y$ is the exact difference in the value of $y$ at $x$ and $x+\Delta y$ while $dy$ is an approx. of the change in $y$ obtained using the slope.

a) $y = \ln\sqrt{x+2} + \ln(3x+1)^3$

$= \frac{1}{2}\ln(x+2) + 3\ln(3x+1)$

$y' = \frac{1}{2(x+2)} + \frac{3}{3x+1} \cdot 3 = \frac{1}{2x+4} + \frac{9}{3x+1}$

$dy = f'(x)dx = f'(x)\Delta x$ So $dy = \left[\frac{1}{2(2)+4} + \frac{9}{3(2)+1}\right](0.009) \doteq 0.0127$

$\Delta y = f(x+\Delta x) - f(x)$

$= f(2.009) - f(2)$

$= \ln\left[\sqrt{2.009+2}\,(3(2.009)+1)^3\right] - \ln\left[\sqrt{2+2}\,(3(2)+1)^3\right]$

$\doteq 6.54355 - 6.53088$

$\doteq 0.0127$

**Bonus.** (5 marks) Evaluate the following:

$$f'(x) = \lim_{h\to 0} \frac{\sqrt{\dfrac{\arctan(7x+7h)\sec(3x+3h)}{e^{5x+5h}}} - \sqrt{\dfrac{\arctan(7x)\sec(3x)}{e^{5x}}}}{h}$$

where $f(x) = \sqrt{\dfrac{\arctan(7x)\sec(3x)}{e^{5x}}}$

So

$$f'(x) = \frac{1}{2}\sqrt{\frac{e^{5x}}{\arctan(7x)\sec(3x)}} \cdot \frac{\left(\frac{1}{1+(7x)^2}\cdot 7\sec(3x) + \arctan(7x)\sec(3x)\tan(3x)\cdot 3\right)e^{5x} - e^{5x}\cdot 5\arctan(7x)\sec(3x)}{(e^{5x})^2}$$