Dawson College: Calculus I (CHEM. TECH.): 201-NYA-05-S04: Winter 2015

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Test 3

This test is graded out of 38 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. Compute the indefinite integral*(1 mark each)*:

a.

$$\int \sec x\,dx = \ln|\sec x + \tan x| + C$$

b.

$$\int \csc x\,dx = -\ln|\csc x + \cot x| + C$$

c.

$$\int \tan x\,dx = -\ln|\cos x| + C$$

d.

$$\int \cot x\,dx = \ln|\sin x| + C$$

e.

$$\int e^x\,dx = e^x + C$$

f.

$$\int \frac{1}{x}\,dx = \ln|x| + C$$

g.

$$\int \cos x\,dx = -\sin x + C$$

Question 2. Compute the indefinite integral.

a. *(2 marks)*

$$\int \frac{x+2}{x^2}\,dx = \int \frac{x}{x^2} + \frac{2}{x^2}\,dx = \int \frac{1}{x} + \frac{2}{x^2}\,dx = \ln|x| - \frac{2}{x} + C$$

b. *(3 marks)*

$$\int \frac{1-\sin x}{1+\cos x}\,dx = \int \frac{(1-\sin x)(1-\cos x)}{(1+\cos x)(1-\cos x)}\,dx = \int \frac{1-\sin x-\cos x+\sin x\cos x}{1-\cos^2 x}\,dx$$

$$= \int \frac{1-\sin x-\cos x+\sin x\cos x}{\sin^2 x}\,dx$$

$$= \int \csc^2 x - \csc x - \cot x\csc x + \cot x\,dx$$

$$= -\cot x + \ln|\csc x+\cot x| + \csc x + \ln|\sin x| + C$$

c. *(3 marks)*

$$\int \frac{e^{\arcsin 2x}}{\sqrt{1-4x^2}}\,dx$$

$u = \arcsin(2x)$

$du = \frac{1}{\sqrt{1-4x^2}}\cdot 2\,dx$

$\frac{du}{2} = \frac{1}{\sqrt{1-4x^2}}\,dx$

$$= \int e^u \frac{du}{2}$$

$$= \frac{e^u}{2} + C$$

$$= \frac{e^{\arcsin(2x)}}{2} + C$$

Question 3. Compute the definite integral:

a. *(4 marks)*

$$\int_0^{\pi/12} \frac{\sec^2 3x}{4+\tan 3x}\,dx = \int_3^5 \frac{1}{u}\frac{du}{3} = \frac{1}{3}\Big[\ln|u|\Big]_3^5$$

$$= \frac{1}{3}\ln 5 - \frac{1}{3}\ln 3$$

$u = 4 + \tan 3x$

$du = \sec^2 3x \cdot 3\,dx$

$\frac{du}{3} = \sec^2 3x \; dx$

$u(\pi/12) = 4 + \tan\frac{3\pi}{12} = 5$

$u(0) = 3 + \tan(3(0)) = 3$

b. *(4 marks)*

$$\int_0^{1/2} \frac{\ln(2x+3)}{2x+3}\,dx = \int_{\ln 3}^{\ln 4} u\,\frac{du}{2} =$$

$u = \ln(2x+3)$

$du = \frac{1}{2x+3} \cdot 2\,dx$

$\frac{du}{2} = \frac{1}{2x+3}\,dx$

$u(1/2) = \ln(2(1/2)+3) = \ln 4$

$u(0) = \ln 3$

$$= \frac{1}{2}\left[\frac{u^2}{2}\right]_{\ln 3}^{\ln 4}$$

$$= \frac{1}{4}\left[(\ln 4)^2 - (\ln 3)^2\right]$$

Question 4. *(5 marks)* Sketch and find the area of the region bounded by the graphs of $y = 4 - x^2$, $y = 4x - x^2$, $x = 0$ and $x = 2$.

$y = 4 - x^2$

y-int: $(0,4)$

x-int: $0 = 4 - x^2$

$x = \pm 2$ $\quad (2,0), (-2,0)$

vertex: $(0,4)$

$y = 4x - x^2$

y-int: $(0,0)$

x-int: $0 = 4x - x^2$

$0 = x(4-x)$

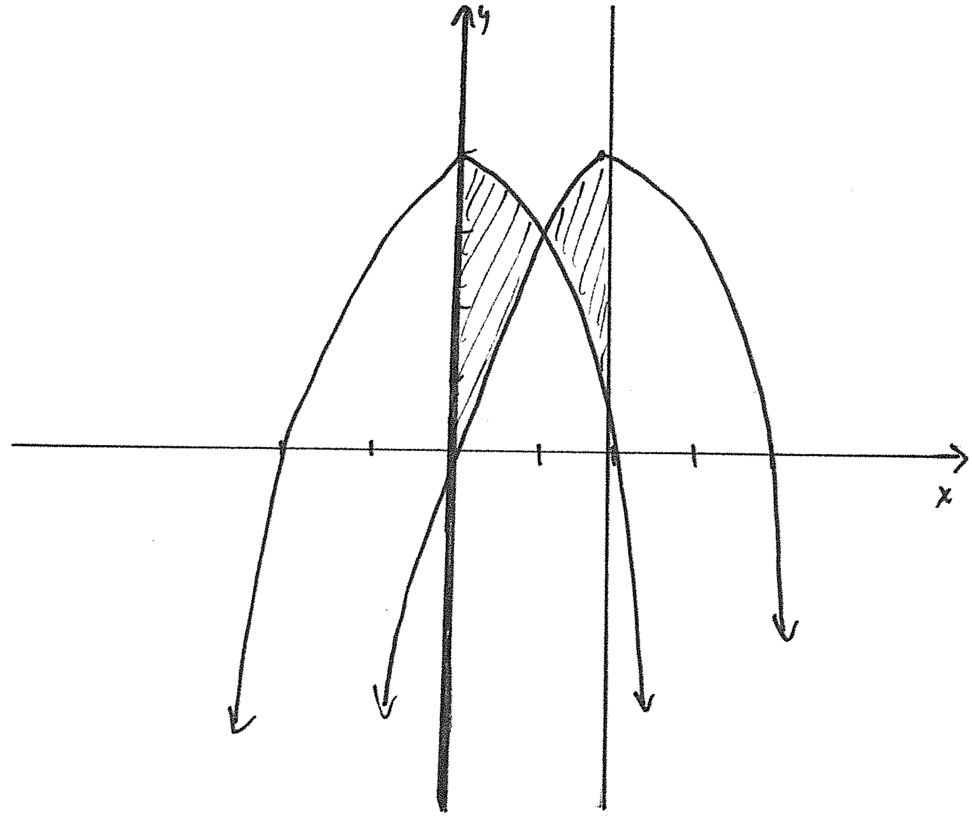
$x = 0 \quad x = 4$

vertex: $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right) = (2,4)$

Intersection of both parabolas

$4 - x^2 = 4x - x^2$

$1 = x$



$$\text{Area} = \int_0^1 4 - x^2 - (4x - x^2)\,dx + \int_1^2 4x - x^2 - (4 - x^2)\,dx$$

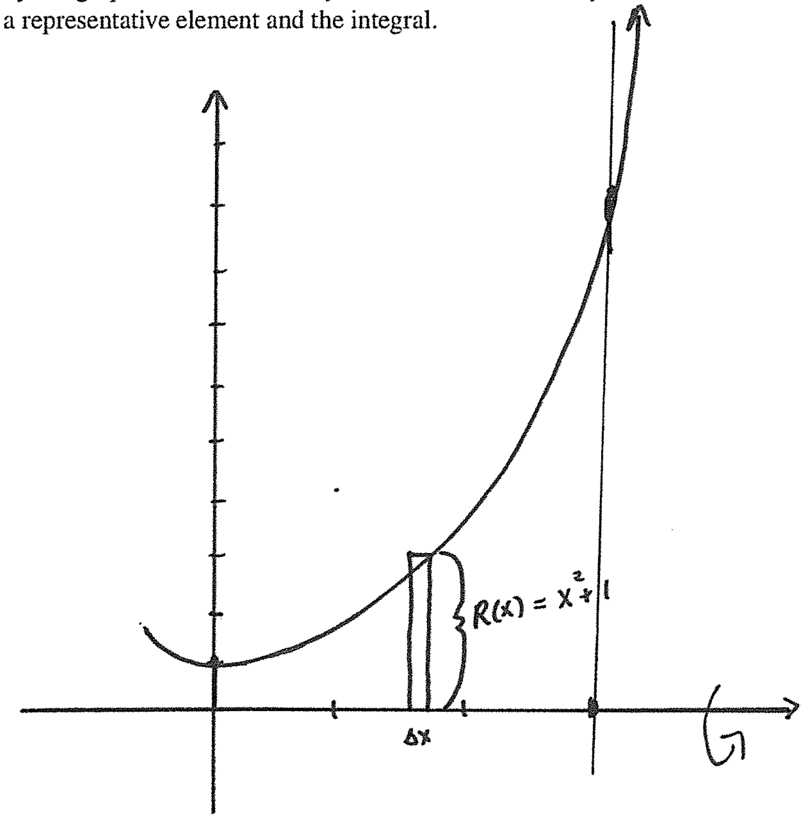
$$= \int_0^1 4 - 4x\,dx + \int_1^2 4x - 4\,dx$$

$$= \left[4x - 2x^2\right]_0^1 + \left[2x^2 - 4x\right]_1^2$$

$$= 4 - 2 + [8 - 8] - [2 - 4]$$

$$= 4$$

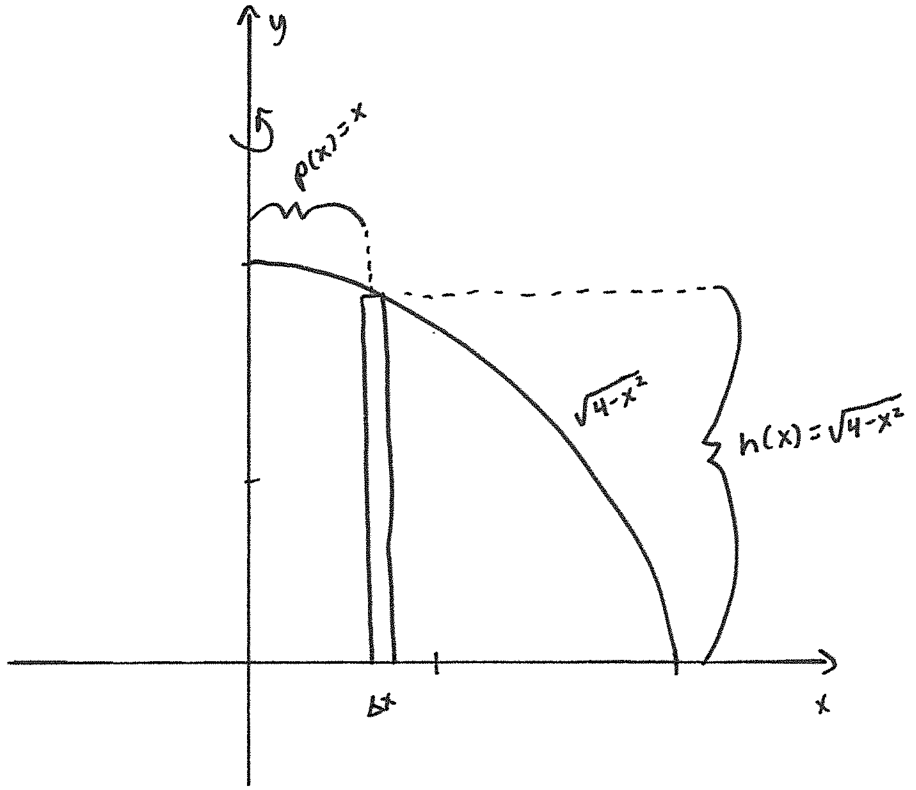
Question 5. *(5 marks)* Using the disk method, set up the integral to find the volume of the solid obtained by rotating the region bounded by the graphs of the functions $y = x^2 + 1$, $x = 0$, $x = 3$, $y = 0$, about the x-axis. Sketch the region, draw a representative rectangle, write a representative element and the integral.



$$\Delta V = \pi (R(x))^2 \Delta x$$
$$= \pi (x^2+1) \Delta x$$

$$V = \int_0^3 \pi (x^2+1)^2 dx$$

Question 6. *(5 marks)* Using the shell method, set up the integral to find the volume of the solid obtained by rotating the region bounded by the graphs of the functions $y = \sqrt{4-x^2}$, in the first quadrant about the y-axis. Sketch the region, draw a representative rectangle, write a representative element and the integral.



$$\Delta V = 2\pi\, p(x)\, h(x)\, \Delta x$$
$$= 2\pi x \sqrt{4-x^2}\, \Delta x$$

$$V = \int_0^2 2\pi x \sqrt{4-x^2}\, dx$$

Bonus. *(5 marks)* Evaluate the definite integral geometrically:

$$\int_1^5 1+2x+3\sqrt{16-(x-5)^2}\, dx = \int_1^5 1+2x\, dx + 3\int_1^5 \sqrt{16-(x-5)^2}\, dx$$

$= \text{area of trapezoid} + 3\left(\text{area of } \tfrac{1}{4} \text{ circle}\right)$

$= 28 + 3(4\pi) = 28 + 12\pi$