

Name: _____
Student ID: _____

Test 2

This test is graded out of 49 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. Compute the derivative of the following functions (*1 mark each*):

a.

$$f(x) = \sin x$$

b.

$$f(x) = \cos x$$

c.

$$f(x) = \csc x$$

d.

$$f(x) = \sec x$$

e.

$$f(x) = \tan x$$

f.

$$f(x) = \cot x$$

g.

$$f(x) = \arcsin x$$

h.

$$f(x) = \arccos x$$

i.

$$f(x) = \arctan x$$

j.

$$f(x) = \ln x$$

k.

$$f(x) = e^x$$

l.

$$f(x) = \operatorname{arcsec} x$$

Question 2. Let $f(x) = x^2(x + 2)^2$

- a. (2 marks) Determine the x and y intercepts of $f(x)$.
- b. (2 marks) Determine $f'(x)$ and solve for the critical points.
- c. (2 marks) State the intervals where $f(x)$ is increasing/decreasing?
- d. (1 mark) Identify the relative minimum and maximum.
- e. (1 mark) Find $f''(x)$.
- f. (2 marks) State the intervals where $f(x)$ is concave up/down?
- g. (1 mark) Identify any inflection points.
- h. (2 marks) Clearly sketch the graph of $f(x)$.

Question 2. Compute the derivative of the following functions. (*Do not simplify.*)

a. (3 marks)

$$f(x) = (\arcsin(2x))(\ln(\sec x))$$

b. (3 marks)

$$g(x) = \frac{x^2 + \sin x}{e^{3x}}$$

c. (3 marks)

$$s(x) = \sqrt{\cot(\arctan(5x))}$$

Question 3. (5 marks) One statement of Boyle's law is that the pressure of a gas varies inversely as the volume for constant temperature. If a Yanngon gas occupies 278 cm^3 when the pressure is 123 kPa and the volume is increasing at the rate of $5.0 \text{ cm}^3/\text{min}$, how fast is the pressure changing when the volume is 314 cm^3 ?

Question 5.¹ (5 marks) A standard soda can is roughly cylindrical and holds 355 cm^3 of liquid. What dimensions should the cylinder be to minimize the material needed to produce the can?

¹from APEX Calculus

Question 6. (3 marks)

a. (3 marks) Find the values of Δy and dy for the given values of x and dx .

$$y = \ln \left[\sqrt{x+2}(3x+1)^3 \right], \quad x = 2, \quad \Delta x = 0.009$$

b. (2 marks) Explain the difference between Δy and dy .

Bonus. (5 marks) Evaluate the following:

$$\lim_{h \rightarrow 0} \frac{\sqrt{\frac{\arctan(7x+7h) \sec(3x+3h)}{e^{5x+5h}}} - \sqrt{\frac{\arctan(7x) \sec(3x)}{e^{5x}}}}{h}$$