

Quiz 10

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. #5.23 Given:

$$\mathcal{L}_1: (x, y, z) = (2, 1, 3) + t(4, 2, -3) \quad t \in \mathbb{R}$$

$$\mathcal{L}_2: (x, y, z) = (3, 1, 2) + t(1, -1, -2) \quad t \in \mathbb{R}$$

- a. (5 marks) Find the point on each line which is closest to the other.
- b. (2 marks) Find the distance between the lines.

a) \mathcal{L}_1 & \mathcal{L}_2 are two skew lines, see notes.

Question 2. (3 marks) §3.5 #34 Prove: If θ is the angle between \vec{u} and \vec{v} and $\vec{u} \cdot \vec{v} \neq 0$, then

$$\tan \theta = \frac{\|\vec{u} \times \vec{v}\|}{\vec{u} \cdot \vec{v}}$$

Demonstrated in class:

$$\|\vec{u} \times \vec{v}\| = \|\vec{u}\| \|\vec{v}\| \sin \theta \quad \Leftrightarrow \quad \sin \theta = \frac{\|\vec{u} \times \vec{v}\|}{\|\vec{u}\| \|\vec{v}\|}$$

$$\vec{u} \cdot \vec{v} = \|\vec{u}\| \|\vec{v}\| \cos \theta$$

$$\cos \theta = \frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\| \|\vec{v}\|}$$

$$\text{LHS} = \tan \theta = \frac{\sin \theta}{\cos \theta} = \frac{\frac{\|\vec{u} \times \vec{v}\|}{\|\vec{u}\| \|\vec{v}\|}}{\frac{\vec{u} \cdot \vec{v}}{\|\vec{u}\| \|\vec{v}\|}} = \frac{\|\vec{u} \times \vec{v}\|}{\vec{u} \cdot \vec{v}}$$