Dawson College: Linear Algebra (SCIENCE): 201-NYC-05-S3: Winter 2015

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Quiz 12

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(5 marks)* §4.2 #2g Use the subspace test to determine which of the following are subspaces of M_{nn}.

The set of all $n \times n$ matrices A such that $AB = BA$ for some fixed $n \times n$ matrix B.

Let $W = \{A \mid A \in M_{n\times n} \text{ and } AB = BA\}$

① Closure under addition

$X, Y \in W \Rightarrow XB = BX$ and $YB = BY$

$X + Y \in W$ since $(X+Y)B = XB + YB$

$= BX + BY$ since $X, Y \in W$

$= B(X+Y)$

② Closure under scalar multiplication

$X \in W$ and $r \in \mathbb{R}$

$rX \in W$ since $(rX)B = rXB$

$= rBX$ since $X \in W$

$= B(rX)$

$\therefore$ W is a subspace of $M_{n\times n}$

Question 2. *(5 marks)* §4.3 #11 Show that if $S = \{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n\}$ is linearly independent set of vectors, then so is every nonempty subset of S.

Suppose there exists a subset $W = \{\vec{v}_{k_1}, \vec{v}_{k_2}, \ldots, \vec{v}_{k_m}\}$ that is linearly dependent $\exists\, c_{k_i} \neq 0$ s.t.

$$\vec{0} = c_{k_1}\vec{v}_{k_1} + c_{k_2}\vec{v}_{k_2} + \cdots + c_{k_i}\vec{v}_{k_i} + \cdots + c_{k_m}\vec{v}_{k_m}$$

Let $S - W = \{\vec{v}_{\ell_1}, \vec{v}_{\ell_2}, \ldots, \vec{v}_{\ell_p}\}$.

Then

$$\vec{0} = c_{k_1}\vec{v}_{k_1} + c_{k_2}\vec{v}_{k_2} + \cdots + c_{k_i}\vec{v}_{k_i} + \cdots + c_{k_m}\vec{v}_{k_m} + c_{\ell_1}\vec{v}_{\ell_1} + c_{\ell_2}\vec{v}_{\ell_2} + \cdots + c_{\ell_p}\vec{v}_{\ell_p}$$

, where $\exists\, c_{k_i} \neq 0$ ↯ since S is linearly independent.

$\therefore$ every nonempty subset of S is linearly independent.