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Quiz 2

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §1.2 #7 *(5 marks)* Solve the given linear system by Gauss-Jordan elimination.

$$\begin{array}{rcrcrcrcl} x & - & y & + & 2z & - & w & = & -1 \\ 2x & + & y & - & 2z & - & 2w & = & -2 \\ -x & + & 2y & - & 4z & + & w & = & 1 \\ 3x & & & & & - & 3w & = & -3 \end{array}$$

$$\begin{bmatrix} 1 & -1 & 2 & -1 & -1 \\ 2 & 1 & -2 & -2 & -2 \\ -1 & 2 & -4 & 1 & 1 \\ 3 & 0 & 0 & -3 & -3 \end{bmatrix}$$

$$\sim \begin{matrix} -2R_1 + R_2 \to R_2 \\ R_1 + R_3 \to R_3 \\ -3R_1 + R_4 \to R_4 \end{matrix} \begin{bmatrix} 1 & -1 & 2 & -1 & -1 \\ 0 & 3 & -6 & 0 & 0 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 3 & -6 & 0 & 0 \end{bmatrix}$$

$$\sim \begin{matrix} \frac{1}{3}R_2 \to R_2 \\ -\frac{1}{3}R_2 + R_3 \to R_3 \\ -R_2 + R_4 \to R_4 \end{matrix} \begin{bmatrix} 1 & -1 & 2 & -1 & -1 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\sim \begin{matrix} R_2 + R_1 \to R_1 \end{matrix} \begin{bmatrix} 1 & 0 & 0 & -1 & -1 \\ 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$z = s \quad s, t \in \mathbb{R}$

$w = t$

$x = -1 + t$

$y = +2s$

$\therefore (x, y, z, w) = (-1+t, 2s, s, t)$

Question 2. §1.2 #26 *(5 marks)* Determine the values of a for which the system has no solutions, exactly one solution, or infinitely many solutions.

$$\begin{array}{rcrcrcl} x & + & 2y & - & z & = & 2 \\ 2x & - & 2y & + & 3z & = & 1 \\ x & + & 2y & - & (a^2-3)z & = & a \end{array}$$

$$\begin{bmatrix} 1 & 2 & -1 & 2 \\ 2 & -2 & 3 & 1 \\ 1 & 2 & -(a^2-3) & a \end{bmatrix}$$

$$\sim \begin{matrix} -2R_1+R_2 \to R_2 \\ -R_1+R_3 \to R_3 \end{matrix} \begin{bmatrix} 1 & 2 & -1 & 2 \\ 0 & -6 & 5 & -3 \\ 0 & 0 & 4-a^2 & a-2 \end{bmatrix}$$

no solution:

Need to find values for 'a' s.t.

$$0x + 0y + (4-a^2)z = a-2$$
$$(4-a^2)z = a-2$$

has no solution if $a = -2$ then

$$0z = -4$$
$$0 = -4 \quad \text{↓ no solution.}$$

unique solution:

var = # leading 1's in var. columns

so $4-a^2 \neq 0$ ∴ $a \neq 2, a \neq -2$

infinitely many solutions:

var > # leading 1's in var. columns

so $a = 2$.