Dawson College: Linear Algebra (SCIENCE): 201-NYC-05-S3: Winter 2015

Name: Y. Lamontagne
Student ID:

Quiz 3

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §1.3 Consider the matrices

$$A=\begin{bmatrix}3&0\\-1&2\\1&1\end{bmatrix},\ B=\begin{bmatrix}4&-1\\0&2\end{bmatrix},\ C=\begin{bmatrix}1&4&2\\3&1&5\end{bmatrix},\ D=\begin{bmatrix}1&5&2\\-1&0&1\\3&2&4\end{bmatrix},\ E=\begin{bmatrix}6&1&3\\-1&1&2\\4&1&3\end{bmatrix}$$

In each part, compute the given expression (where possible).

#51. *(3 marks)* $\mathrm{tr}((EC^T)^TA)$

$$C^T=\begin{bmatrix}1&3\\4&1\\2&5\end{bmatrix}$$

$$EC^T$$

$$=\begin{bmatrix}6&1&3\\-1&1&2\\4&1&3\end{bmatrix}\begin{bmatrix}1&3\\4&1\\2&5\end{bmatrix}$$

$$=\begin{bmatrix}16&34\\7&8\\14&28\end{bmatrix}$$

$$(EC^T)^TA=\begin{bmatrix}16&7&14\\34&8&28\end{bmatrix}\begin{bmatrix}3&0\\-1&2\\1&1\end{bmatrix}$$

$$=\begin{bmatrix}55&28\\122&44\end{bmatrix}$$

$$\mathrm{tr}((EC^T)^TA)=55+44=99$$

Question 2. §1.3 #21 *(2 marks)* Prove: If A and B are $n\times n$ matrices, then $\mathrm{tr}(A+B)=\mathrm{tr}(A)+\mathrm{tr}(B)$

Let $A=[a_{ij}]_{nn}$

$B=[b_{ij}]_{nn}$

$$\mathrm{tr}(A+B)=\mathrm{tr}([a_{ij}]_{nn}+[b_{ij}]_{nn})$$

$$=\mathrm{tr}([a_{ij}+b_{ij}]_{nn})$$

$$=(a_{11}+b_{11})+(a_{22}+b_{22})+\dots+(a_{nn}+b_{nn})$$

$$=(a_{11}+a_{22}+\dots+a_{nn})+(b_{11}+b_{22}+\dots+b_{nn})$$

$$=\mathrm{tr}(A)+\mathrm{tr}(B)$$

Question 3. §1.3 #29 A matrix B is said to be a *square root* of a matrix A if $BB = A$.

a. *(2 marks)* Find two square roots of $A = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$.

b. *(3 marks)* How many different square roots can you find of $A = \begin{bmatrix} 5 & 0 \\ 0 & 9 \end{bmatrix}$

Let $B = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $BB = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$= \begin{bmatrix} a^2 + cb & ab + bd \\ ac + dc & bc + d^2 \end{bmatrix}$$

a) $BB = \begin{bmatrix} 2 & 2 \\ 2 & 2 \end{bmatrix}$ we get

$$a^2 + cb = 2$$
$$ab + bd = 2$$
$$ac + dc = 2$$
$$bc + d^2 = 2$$

note that if $a = b = c = d = 1$ or $a = b = c = d = -1$ it satisfies the above

b) $BB = \begin{bmatrix} 5 & 0 \\ 0 & 9 \end{bmatrix}$ we get

$$a^2 + cb = 5$$
$$ab + bd = 0 \Leftrightarrow b(a+d) = 0$$
$$ac + dc = 0 \Leftrightarrow c(a+d) = 0$$
$$bc + d^2 = 9$$

if $b = 0$ then $a^2 = 5$, $d^2 = 9$; $a = \pm\sqrt{5}$, $d = \pm 3$ so 4 matrices

$\Rightarrow a + d \neq 0$

$\Rightarrow c = 0$

similarly if $c = 0$

if $a = -d$ then $(-d)^2 + cb = 5$

$d^2 + cb = 5$ and $bc + d^2 = 9$ ↯

$\therefore$ only 4 matrices.