Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S3: Winter 2015

Name: ____________________
Student ID: ____________________

Quiz 4

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §1.4 # 18 Let A be the matrix

$$A = \begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix}$$

In each part, compute the given quantity.

a. *(2 marks)* A^3

b. *(2 marks)* A^{-3}

f. *(1 mark)* $p(A)$, where $p(x) = x^3 - 2x + 4$

a) $A^3 = AAA = \begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix} = \begin{bmatrix} 4 & 0 \\ 12 & 1 \end{bmatrix}\begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix}$

$= \begin{bmatrix} 8 & 0 \\ 28 & 1 \end{bmatrix}$

b) $A^{-1} = \dfrac{1}{8(1) - 28(0)} \begin{bmatrix} 1 & 0 \\ -28 & 8 \end{bmatrix}$

$= \begin{bmatrix} 1/8 & 0 \\ -7/2 & 1 \end{bmatrix}$

f) $p(A) = A^3 - 2A + 4I$

$= \begin{bmatrix} 8 & 0 \\ 28 & 1 \end{bmatrix} - 2\begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix} + 4\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 8 & 0 \\ 20 & 3 \end{bmatrix}$

Question 2. *(3 marks)* §1.5 #29a Show that a matrix with a row of zeros cannot have an inverse.

Let the i^{th} row of A be zeros. Then if B is any matrix

$[i^{th}$ row of $AB] = [i^{th}$ row of $A]\,B = [0\ 0\ \dots\ 0]$

So impossible to get the identity.

$\therefore$ no inverse

Question 3. *(2 marks)* §1.3 #21 Show that if A is invertible and k is any nonzero scalar then $(kA)^n = k^nA^n$ for all integer values of n.

if $n=0$ $\quad (KA)^0 = K^0A^0 \iff I = 1\cdot I$ since $K \neq 0$ and A is invertible

if $n \in \mathbb{Z}^+$ $\quad (KA)^n = \underbrace{KA\,KA \dots KA}_{n \text{ times}} = \underbrace{K\cdot K \cdots K}_{n \text{ times}}\ \underbrace{AA\dots A}_{n \text{ times}} = K^nA^n$

if $n \in \mathbb{Z}^-$ $\quad (KA)^n = ((KA)^{-1})^{-n} = (\tfrac{1}{K}A^{-1})^{-n} = \underbrace{\tfrac{1}{K}A^{-1}\tfrac{1}{K}A^{-1} \dots \tfrac{1}{K}A^{-1}}_{-n \text{ times}} = \underbrace{\tfrac{1}{K}\tfrac{1}{K}\dots\tfrac{1}{K}}_{-n \text{ times}}\ \underbrace{A^{-1}A^{-1}\dots A^{-1}}_{-n \text{ times}}$

$= (\tfrac{1}{K})^{-n}(A^{-1})^{-n} = K^nA^n$