Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S3: Winter 2015

Name: Y. Lamontagne
Student ID:

Quiz 5

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(5 marks)* §1.5 #21 Use the inversion algorithm to find the inverse of the given matrix, if the inverse exists.

$$A = \begin{bmatrix} 2 & -4 & 0 & 0 \\ 1 & 2 & 12 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & -1 & -4 & -5 \end{bmatrix} \quad [A|I] = \left[\begin{array}{cccc|cccc} 2 & -4 & 0 & 0 & 1 & 0 & 0 & 0 \\ 1 & 2 & 12 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & -4 & -5 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$\underset{R_1 \leftrightarrow R_2}{\sim} \left[\begin{array}{cccc|cccc} 1 & 2 & 12 & 0 & 0 & 1 & 0 & 0 \\ 2 & -4 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & -4 & -5 & 0 & 0 & 0 & 1 \end{array}\right] \underset{-2R_1+R_2 \to R_2}{\sim} \left[\begin{array}{cccc|cccc} 1 & 2 & 12 & 0 & 0 & 1 & 0 & 0 \\ 0 & -8 & -24 & 0 & 1 & -2 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 1 & 0 \\ 0 & -1 & -4 & -5 & 0 & 0 & 0 & 1 \end{array}\right]$$

$$\underset{\substack{-6R_3+R_1 \to R_1 \\ 12R_3+R_2 \to R_2 \\ -8R_4 \to R_4}}{\sim} \left[\begin{array}{cccc|cccc} 1 & 2 & 0 & 0 & 0 & 1 & -6 & 0 \\ 0 & -8 & 0 & 0 & 1 & -2 & 12 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 1 & 0 \\ 0 & 8 & 32 & 40 & 0 & 0 & 0 & -8 \end{array}\right] \underset{\substack{4R_1 \to R_1 \\ R_2+R_4 \to R_4}}{\sim} \left[\begin{array}{cccc|cccc} 4 & 8 & 0 & 0 & 0 & 4 & -24 & 0 \\ 0 & -8 & 0 & 0 & 1 & -2 & 12 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 32 & 40 & 1 & -2 & 12 & -8 \end{array}\right]$$

$$\underset{\substack{R_2+R_1 \to R_1 \\ -16R_3+R_4 \to R_4}}{\sim} \left[\begin{array}{cccc|cccc} 4 & 0 & 0 & 0 & 1 & 2 & -12 & 0 \\ 0 & -8 & 0 & 0 & 1 & -2 & 12 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 40 & 1 & -2 & -4 & -8 \end{array}\right] \underset{\substack{\frac{1}{4}R_1 \to R_1 \\ -\frac{1}{8}R_2 \to R_2 \\ \frac{1}{2}R_3 \to R_3 \\ \frac{1}{40}R_4 \to R_4}}{\sim} \left[\begin{array}{cccc|cccc} 1 & 0 & 0 & 0 & 1/4 & 1/2 & -3 & 0 \\ 0 & 1 & 0 & 0 & -1/8 & 1/4 & -3/2 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 1/2 & 0 \\ 0 & 0 & 0 & 1 & 1/40 & -1/20 & -\frac{1}{10} & -1/5 \end{array}\right]$$

Question 2. *(5 marks)* §1.5 #42 Prove that if A is an invertible matrix and B is row equivalent to A, then B is also invertible.

Premise: ① A invertible
② B row equivalent to A

WTS: B is invertible

By TFAE it is sufficient to show that B can be expressed as a product of elementary matrices.

From ① and TFAE $\exists\, E_1, \ldots, E_k$ s.t.

$A = E_1 \cdots E_k$ and E_i are elem. matrices

From ② $\exists\, F_1, \ldots, F_\ell$ s.t.

$F_\ell \cdots F_1 B = A$ where F_i are elem. matrices

$$F_\ell \ldots F_1 B = A$$
$$F_\ell \ldots F_1 B = E_1 \ldots E_k$$
$$(F_\ell \ldots F_1)^{-1} F_\ell \ldots F_1 B = (F_\ell \ldots F_1)^{-1} E_1 \ldots E_k$$
$$I B = F_1^{-1} \ldots F_\ell^{-1} E_1 \ldots E_k$$
$$B = F_1^{-1} \ldots F_\ell^{-1} E_1 \ldots E_k$$

where F_i^{-1} are elem. matrices since inverse of elem. matrices are elem. matrices.