Dawson College: **Linear Algebra** (SCIENCE): 201-NYC-05-S3: Winter 2015

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Quiz 6

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(3 marks)* §1.6 # T-F a) It is impossible for a system of linear equations to have exactly two solutions.

True, Let x_1 and x_2 be solutions to the system $Ax=b$ s.t. $x_1 \neq x_2$. Then $x = x_1 + k(x_2 - x_1)$ $\forall k \in \mathbb{R}$ are solution to $Ax=b$. Since

$$\begin{aligned} Ax = A(x_1 + k(x_2-x_1)) &= Ax_1 + kA(x_2 - x_1) \\ &= b + k(Ax_2 - Ax_1) \\ &= b + k(b-b) \\ &= b + k(0) = b. \end{aligned}$$

Question 2. *(2 marks)* §1.6 #13 Determine conditions on the $b_i's$, if any, in order to guarantee that the linear system is consistent.

$$\begin{aligned} 6x_1 - 4x_2 &= b_1 \\ 3x_1 - 2x_2 &= b_2 \end{aligned}$$

$$\begin{bmatrix} 6 & -4 & b_1 \\ 3 & -2 & b_2 \end{bmatrix} \sim_{\frac{1}{2}R_1 \to R_1} \begin{bmatrix} 3 & -2 & \frac{b_1}{2} \\ 3 & -2 & b_2 \end{bmatrix} \sim_{-R_1+R_2 \to R_2} \begin{bmatrix} 3 & -2 & b_1/2 \\ 0 & 0 & b_2 - \frac{b_1}{2} \end{bmatrix}$$

To be consistent $b_2 - \frac{b_1}{2} = 0$

$2b_2 = b_1$

Question 3. *(3 marks)* §1.7 #33 Prove: If $A^TA = A$, then A is symmetric and $A = A^2$.

Premise: $A^TA = A$

WTS: $\cdot\ A^T = A$
$\cdot\ A^2 = A$

$$\begin{aligned} LHS &= A^T \\ &= (A^TA)^T \\ &= A^T(A^T)^T = A^TA = A = RHS \end{aligned}$$

$$\begin{aligned} LHS &= A^2 \\ &= AA \\ &= A^TA \quad \text{since } A \text{ was shown to be symmetric} \\ &= A = RHS \end{aligned}$$

Question 4. *(2 marks)* §1.7 #26 Find all values of x in order for A to be invertible

$$A = \begin{bmatrix} x-\frac{1}{2} & 0 & 0 \\ x & x-\frac{1}{3} & 0 \\ x^2 & x^3 & x+\frac{1}{4} \end{bmatrix}$$

a triangular or diagonal matrix is invertible if the element of the main diagonal are all non-zero. So $x \neq \frac{1}{2}, \frac{1}{3}, -\frac{1}{4}$

$\therefore$ A is invertible if $x \in \mathbb{R} \setminus \{\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}\}$