Dawson College: Linear Algebra (SCIENCE): 201-NYC-05-S3: Winter 2015

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Quiz 7

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. *(4 marks)* §2.1 #40 Prove that (x_1,y_1), (x_2,y_2), and (x_3,y_3) are collinear points if and only if

$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0.$$

[⇒] if (x_1,y_1), (x_2,y_2) & (x_3,y_3) are collinear then

$$y_1 = mx_1 + b$$
$$y_2 = mx_2 + b$$
$$y_3 = mx_3 + b$$

then $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

$$= \begin{vmatrix} x_2 & y_2 \\ x_3 & y_3 \end{vmatrix} - \begin{vmatrix} x_1 & y_1 \\ x_3 & y_3 \end{vmatrix} + \begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix}$$

$$= x_2y_3 - y_2x_3 - x_1y_3 + x_3y_1 + x_1y_2 - y_1x_2$$

$$= x_2(mx_3+b) - (mx_2+b)x_3 - x_1(mx_3+b) + x_3(mx_1+b) + x_1(mx_2+b) - (mx_1+b)x_2$$

$$= \cancel{mx_2x_3} + \cancel{x_2b} - \cancel{mx_2x_3} - \cancel{bx_3} - \cancel{mx_1x_3} - \cancel{x_1b} + \cancel{mx_1x_3} + \cancel{bx_3} + \cancel{mx_1x_2} + \cancel{bx_1} - \cancel{mx_1x_2} - \cancel{bx_2}$$

$$= 0$$

[⇐] if $\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$ then one column is a linear combination of the others (wolog), the second column

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} = m \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} + b \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$\therefore$
$$y_1 = mx_1 + b$$
$$y_2 = mx_2 + b$$
$$y_3 = mx_3 + b$$

$\therefore$ (x_1,y_1), (x_2,y_2), (x_3,y_3) are collinear.

Question 2. *(3 marks)* §2.2 #13 Evaluate the determinant of the given matrix by reducing the matrix to row echelon form.

$$A = \begin{bmatrix} 3 & -6 & 9 \\ -2 & 7 & -2 \\ 0 & 1 & 5 \end{bmatrix} \sim_{\frac{1}{3}R_1 \to R_1} \begin{bmatrix} 1 & -2 & 3 \\ -2 & 7 & -2 \\ 0 & 1 & 5 \end{bmatrix} \sim_{2R_1+R_2 \to R_2} \begin{bmatrix} 1 & -2 & 3 \\ 0 & 3 & 4 \\ 0 & 1 & 5 \end{bmatrix}$$

$$\sim_{R_2 \leftrightarrow R_3} \begin{bmatrix} 1 & -2 & 3 \\ 0 & 1 & 5 \\ 0 & 3 & 4 \end{bmatrix} \sim_{-3R_2+R_3 \to R_3} \begin{bmatrix} 1 & -2 & 3 \\ 0 & 1 & 5 \\ 0 & 0 & -11 \end{bmatrix} = B$$

$$\left(\tfrac{1}{3}\right)(-1)\det A = \det B$$

$$\det A = 33.$$

Question 3. *(3 marks)* §2.3 #33 Prove that if $\det(A) = 1$ and all the entries in A are integers, then all the entries in A^{-1} are integers.

$$A^{-1} = \frac{1}{\det A} \operatorname{adj} A$$

$$= \frac{1}{1} \operatorname{adj} A$$

$$= \operatorname{adj} A$$

Since the adj A is generated by cofactors. Cofactors are ultimately the product, sum and difference of the entries of the matrix A. The product, sum and difference of integers are integers.

$\therefore$ the entries of A^{-1} are integers.

Question 4. The augmented matrix of a linear system is given by

$$\begin{bmatrix} 1 & 2 & 3 & 4 & \pi \\ 0 & \sqrt{2} & 4 & 5 & 6 \\ 0 & 0 & 0 & a & b \end{bmatrix}$$

If possible for what values of a and b there is

a. *(2 marks)* no solution? Justify.

b. *(2 marks)* exactly one solution? Justify.

c. *(1 mark)* infinitely many solutions?

See Test #1