

Quiz 8

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. (2 marks) §3.1 #TF a) Determine whether the statement is true or false, and justify your answer. Two equivalent vectors must have the same initial point.

False, By definition two vectors are equivalent iff they have the same direction and magnitude. The initial point does not need to be the same.

Question 2. (3 marks) §3.1 #25 Let $\vec{u} = (1, -1, 3, 5)$ and $\vec{v} = (2, 1, 0, -3)$. Find scalars a and b so that $a\vec{u} + b\vec{v} = (1, -4, 9, 18)$.

$$a(1, -1, 3, 5) + b(2, 1, 0, -3) = (1, -4, 9, 18)$$

$$\begin{aligned} a(1) + b(2) &= 1 \\ a(-1) + b(1) &= -4 \\ a(3) + b(0) &= 9 \Rightarrow a = 3 \\ a(5) + b(-3) &= 18 \Rightarrow 3(5) + b(-3) = 18 \\ &\quad (-3)b = 3 \\ &\quad b = -1 \end{aligned}$$

verify:

$$3(-1) + (-1)(1) = -4 \quad \checkmark$$

$$3(1) + (-1)(2) = 1 \quad \checkmark$$

Determine whether the statement is true or false, and justify your answer.

Question 3. (2 marks) §3.2 #TF e) If $\|\vec{u}\| = 2$, $\|\vec{v}\| = 1$ and $\vec{u} \cdot \vec{v} = 1$, then the angles between \vec{u} and \vec{v} is $\frac{\pi}{3}$ radians.

True,

$$\begin{aligned} \text{Since } \vec{u} \cdot \vec{v} &= \|\vec{u}\| \|\vec{v}\| \cos \theta \\ 1 &= 2 \cdot 1 \cdot \cos \theta \\ \cos \theta &= \frac{1}{2} \end{aligned}$$

$$\theta = \pi/3$$

Question 4. (2 marks) §3.2 #27 Let $p_0 = (x_0, y_0, z_0)$ and $p = (x, y, z)$ Describe the set of all points (x, y, z) for which $\|p - p_0\| = 1$.

$$\begin{aligned} \|p - p_0\| &= 1 \\ 1 &= \|(x - x_0, y - y_0, z - z_0)\| \\ 1 &= \sqrt{(x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2} \\ 1 &= (x - x_0)^2 + (y - y_0)^2 + (z - z_0)^2 \end{aligned}$$

A sphere of radius 1 centered at (x_0, y_0, z_0) .

Question 5. (1 mark) §3.2 #19a Find a unit vector that has the same direction as the given vector.

$$\begin{aligned} \vec{v} &= (-4, -3) \\ \vec{u} &= \frac{\vec{v}}{\|\vec{v}\|} = \frac{(-4, -3)}{\sqrt{(-4)^2 + (-3)^2}} = \frac{(-4, -3)}{\sqrt{25}} = \frac{(-4, -3)}{5} \\ &= \left(\frac{-4}{5}, \frac{-3}{5}\right) \end{aligned}$$