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Quiz 11

This quiz is graded out of 10 marks. No books, calculators, notes or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. §4.1 #11 Determine whether each set equipped with the given operations is a vector space. For those that are not vector spaces identify the vector space axioms that fail.

The set of all pairs of real numbers of the form $(1,x)$ with the operations

$(1,y)+(1,y')=(1,y+y')$ and $k(1,y)=(1,ky)$ Let $\vec{u}=(1,u)$, $\vec{v}=(1,v)$, $\vec{w}=(1,w)$ $r,s\in\mathbb{R}$

① closure under $+$: $\vec{u}+\vec{v}=(1,u)+(1,v)=(1,u+v)$, closed since $u+v\in\mathbb{R}$

② $+$ is commutative: $\vec{u}+\vec{v}=(1,u)+(1,v)=(1,u+v)=(1,v+u)=(1,v)+(1,u)=\vec{v}+\vec{u}$

③ $+$ is associative: $(\vec{u}+\vec{v})+\vec{w}=((1,u)+(1,v))+(1,w)=(1,u+v)+(1,w)$
$=(1,(u+v)+w)$ since $\mathbb{R}+$ is
$=(1,u+(v+w))$ associative.
$=(1,u)+(1,v+w)$
$=(1,u)+((1,v)+(1,w))$
$=\vec{u}+(\vec{v}+\vec{w})$

④ $\vec{0}=(1,0)$ is an element of the set since $0\in\mathbb{R}$
and $\vec{u}+\vec{0}=(1,u)+(1,0)=(1,u)=\vec{u}$

⑤ the additive inverse of $\vec{u}=(1,u)$ exists since $(1,-u)$ is an element of the set because $-u\in\mathbb{R}$. And $\vec{u}+(1,-u)=(1,u)+(1,-u)$
$=(1,0)$
$=\vec{0}$

⑥ $r\cdot\vec{u}=r(1,u)=(1,ru)$, closed under $\cdot$ since $ru\in\mathbb{R}$.

⑦ distributivity over $\cdot$: $(r+s)\vec{u}=(r+s)(1,u)=(1,(r+s)u)=(1,ru+su)$
$=(1,ru)+(1,su)$
$=r(1,u)+s(1,u)$
$=r\vec{u}+s\vec{u}$

⑧ distributivity over: $r(\vec{u}+\vec{v})=r((1,u)+(1,v))$
$=r(1,u+v)$
$=(1,r(u+v))=(1,ru+rv)$
$=(1,ru)+(1,rv)$
$=r(1,u)+r(1,v)=r\vec{u}+r\vec{v}$

⑨ $\cdot$ is associative
$(rs)\cdot\vec{u}$
$=(rs)(1,u)$
$=(1,(rs)u)$
$=r(1,su)$
$=r(s(1,u))$
$=r(s\vec{u})$

⑩ $1\cdot\vec{u}=1\cdot(1,u)=(1,1\cdot u)=(1,u)=\vec{u}$

$\therefore$ The above is a vectorspace.