Dawson College: **Linear Algebra**: 201-NYC-05-S03: Winter 2015

Name: ____________________
Student ID: ____________________

Test 1

This test is graded out of 48 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1.

a. *(6 marks)* Solve the following system by Gauss-Jordan elimination:

$$\begin{array}{rcrcrcrcrcrcr} 3x_1 & - & 2x_2 & + & x_3 & - & 3x_4 & + & x_5 & - & 4x_6 & = & 0 \\ 4x_1 & + & 3x_2 & - & x_3 & + & x_4 & - & 2x_5 & - & 3x_6 & = & 0 \\ 7x_1 & + & x_2 & & & - & 2x_4 & - & x_5 & + & 5x_6 & = & 0 \end{array}$$

b. *(1 mark)* Find a particular nontrivial solution to the above system.

c. *(1 mark)* Find a solution to the above system where $x_1 = 1$.

Question 2. Consider the matrices:

$$A = \begin{bmatrix} 2 & 2 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 2 \end{bmatrix}, B = \begin{bmatrix} 2 & 0 & 1 \\ -3 & -4 & 0 \end{bmatrix} C = \begin{bmatrix} 2 & 1 \\ 0 & 2 \\ 3 & -1 \end{bmatrix} D = \begin{bmatrix} 2 & 1 \\ 1 & -2 \end{bmatrix} F = \begin{bmatrix} 5 & -1 \\ 0 & -2 \\ 1 & 0 \end{bmatrix}$$

a. *(2 marks)* Compute the following, if possible.

$$FAB$$

b. *(2 marks)* Compute the following, if possible.

$$F^T AB^T$$

c. *(2 marks)* Compute the following, if possible.

$$(CD^{-1})^T$$

d. *(5 marks)* Find E, if possible.

$$(I - E^T)^{-1} = F^T AB^T - 12I$$

Question 3. Consider

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 7 \\ 3 & 2 & 0 \end{bmatrix}.$$

a. *(5 marks)* Find A^{-1}.

b. *(2 marks)* Determine $\text{tr}(A^{-1})$ and $\text{tr}(A^{-1}A)$.

c. *(3 marks)* Using a. solve $Ax = b$ where $x = [x_1, x_2, x_3]^T$ and $b = [-1, -2, 3]^T$.

Question 4. *(4 marks)* Let A be a square matrix. If B is a square matrix satisfying $AB = I$, then $B = A^{-1}$.

Question 5. *(4 marks)* Let $X = A^T A + B$ be a square matrix. Prove: If $B^2 = 0$, $AB = BA^T = 0$ then X^2 is symmetric.

Question 6. *(5 marks)* Show that

$$A = \begin{bmatrix} 5 & 7 & 9 \\ 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

and

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 8 & 10 & 12 \end{bmatrix}$$

are row-equivalent by finding 3 elementary matrices E_i such that $E_3E_2E_1A = B$.

Question 7. The augmented matrix of a linear system is given by

$$\begin{bmatrix} 1 & 2 & 3 & 4 & \pi \\ 0 & \sqrt{2} & 4 & 5 & 6 \\ 0 & 0 & 0 & a & b \end{bmatrix}$$

If possible for what values of a and b there is

a. *(2 marks)* no solution? Justify.

b. *(2 marks)* exactly one solution? Justify.

c. *(2 marks)* infinitely many solutions? Justify.

Bonus Question.[1] *(5 marks)*

"This system of n linear equations with n unknowns," said the Great Mathematician, "as a curious property."

"Good heavens!" said the Physics Teacher, "What is it?"

"Note," said the Great Mathematician, "that the constants are in arithmetic progression."

"It's all so clear when you explain it!" said the Physics Teacher. "Do you mean like $6x+9y=12$ and $15x+18y=21$?"

"Quite so," said the Great Mathematician, pulling out his bassoon. "Indeed, such a system has a unique solution for some n. Can you find them?"

"Good heavens!" cried the Physics Teacher, "I am baffled"

Are you?

[1] modified from Underwood Dudley, Arnold Lebow (proposers), David Rothman (solver), Elementary problem 1151, American Mathematical Monthly, vol. 70 no. 1 (Jan. 1963), p. 93.