Dawson College: **Linear Algebra**: 201-NYC-05-S03: Winter 2015

Name: ____________________
Student ID: ____________________

Test 2

This test is graded out of 47 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. Given

$$
\begin{array}{rrcll}
\mathscr{A}: & (1,-2,3) & & & \\
\mathscr{L}_1: & (x,y,z) & = & (1+t, 3+2t, 5-t) & t\in\mathbb{R} \\
\mathscr{L}_2: & (x,y,z) & = & (2+2s, 4, 6-3s) & s\in\mathbb{R} \\
\mathscr{P}_1: & x-2y-3z-\pi & = & 0 &
\end{array}
$$

a. *(1 mark)* Determine the equation of the line $\mathscr{L}_3$ that passes through $\mathscr{A}$ and is perpendicular to $\mathscr{P}_1$.

b. *(2 marks)* Determine the point on $\mathscr{P}_1$ that is closest to $\mathscr{A}$.

c. *(3 marks)* Determine the point on $\mathscr{L}_1$ that is closest to $\mathscr{A}$.

d. *(2 marks)* Are $\mathscr{P}_1$ and $\mathscr{L}_1$ parallel, perpendicular, or neither, justify?

e. *(3 marks)* Are $\mathscr{P}_1$ and $\mathscr{L}_2$ parallel, perpendicular, or neither, justify? Find the shortest distance between $\mathscr{P}_1$ and $\mathscr{L}_2$.

Question 2. Given

$$A = \begin{bmatrix} 10 & 1 & 1 & 1 & 9 & 3 & 4 & 1 & 0 & 9 \\ 0 & 9 & 1 & 1 & 4 & 9 & 2 & 7 & 7 & 9 \\ 0 & 0 & 8 & 1 & 1 & 4 & 9 & 2 & 7 & 7 \\ 0 & 0 & 0 & 7 & 1 & 1 & 4 & 9 & 2 & 7 \\ 0 & 0 & 0 & 0 & 6 & 1 & 1 & 4 & 9 & 2 \\ 0 & 0 & 0 & 0 & 0 & 5 & 1 & 1 & 4 & 9 \\ 0 & 0 & 0 & 0 & 0 & 0 & 4 & 1 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 1 & 2 & 6 \\ 2 & 3 & 3 & -3 \\ -3 & 3 & 1 & 9 \\ 0 & 1 & -1 & -12 \end{bmatrix}, \quad C = \begin{bmatrix} -1 & 3 & 2 & 3 \\ 2 & 0 & 3 & 0 \\ -4 & 0 & 1 & 2 \\ 1 & 5 & 0 & -3 \end{bmatrix}$$

a. *(4 marks)* Evaluate $\det(2\,\mathrm{adj}(A) + 3A^{-1})$, if possible.

b. *(4 marks)* If D is a 4×4 matrix and $B^{-1}D^2 = I$ then determine $\det(D)$, if possible.

c. *(4 marks)* Compute $\det(C)$.

Question 3.[1] *(2 marks)* Prove or disprove: If N is a matrix such that $\det(N) = 1$ then $N^2 = I$

Question 4. *(3 marks)* Consider

$$\begin{array}{rcrcrcr} 3x_1 & - & x_2 & + & x_3 & = & 1 \\ & & \alpha x_2 & - & \beta x_3 & = & 1 \\ & & 2x_2 & - & x_3 & = & -1 \end{array}$$

where the determinant of the coefficient matrix is 1 and the x_2 component of the solution is 1. Determine α and β, if possible. *(hint: use Cramer's rule)*

Question 5. §3.3 # 21

a. *(2 marks)* The equation $3x + 2y + z = 6$ can be viewed as a linear system of one equation in three unknowns. Express a general solution of this equation as a particular solution plus a general solution of the associated homogeneous system.

b. *(1 mark)* Give a geometric interpretation of the result in part a.

[1] Modified from a John Abbott final examination problem

Question 7.[2] A triangle is created by joining the $x-$, $y-$, and $z-$intercepts of the plane x + 2y + 3z = 12.

a. *(1 mark)* Sketch the plane.

b. *(1 mark)* Find the coordinates of the vertices of the triangle.

c. *(2 marks)* Find the area of the triangle.

d. *(3 marks)* If θ is the angle between the plane x + 2y + 3z = 12 and the xy-plane, find the value of $\cos\theta$.

[2]Modified from a John Abbott final examination problem

Bonus Question. *(5 marks)* CAUCHY-SCHWARTZ INEQUALITY: If $\vec{u}, \vec{v} \in \mathbb{R}^n$ then $|\vec{u} \cdot \vec{v}| \leq ||\vec{u}||||\vec{v}||$. Prove the CAUCHY-SCHWARTZ INEQUALITY without assuming that the law of cosine holds in $\mathbb{R}^n$. *(hint: look at the squared norm of $||\vec{u}||\vec{v} - ||\vec{v}||\vec{u}$ and $||\vec{u}||\vec{v} + ||\vec{v}||\vec{u}$)*