Dawson College: **Linear Algebra**: 201-NYC-05-S03: Winter 2015

Name: ____________________
Student ID: ____________________

Test 3

This test is graded out of 38 marks. No books, notes, graphing calculators or cell phones are allowed. You must show all your work, the correct answer is worth 1 mark the remaining marks are given for the work. If you need more space for your answer use the back of the page.

Question 1. Given

$$\begin{array}{llcll} \mathscr{L}_1: & (x,y,z) & = & (1+t,3+2t,5-t) & t\in\mathbb{R} \\ \mathscr{L}_2: & (x,y,z) & = & (2+2s,4,6-3s) & s\in\mathbb{R} \end{array}$$

a. *(5 marks)* Are $\mathscr{L}_1$ and $\mathscr{L}_2$ parallel, perpendicular, or neither, justify? Find the points on $\mathscr{L}_1$ and $\mathscr{L}_2$ which are closest to each other.

b. *(2 marks)* Find the shortest distance between $\mathscr{L}_1$ and $\mathscr{L}_2$.

Question 2.[1] Let $\mathscr{H} = \{X \mid X \in \mathscr{M}_{2\times2} \text{ and } AX - X = 0\}$ where $A = \begin{bmatrix} 2 & -2 \\ 2 & -3 \end{bmatrix}$ and with the usual addition and scalar multiplication.

a. *(2 marks)* Give an example of a non-zero matrix in $\mathscr{H}$. Justify.

b. *(2 marks)* Does $\mathscr{H}$ satisfy closure under vector addition? Justify.

c. *(2 marks)* Does $\mathscr{H}$ contain the zero vector of $\mathscr{M}_{2\times2}$ (the vector space of 2×2 matrices)? Justify.

d. *(2 marks)* Does $\mathscr{H}$ satisfy closure under scalar multiplication? Justify.

e. *(2 marks)* Is $\mathscr{H}$ a vector subspace of $\mathscr{M}_{2\times2}$ (the vector space of 2×2 matrices)? Justify.

[1] From a John Abbott final examination

Question 3. *(2 marks)* Determine whether the following is a vector space:

$$\mathscr{Y} = \{(1, y) \mid y \in \mathbb{R}\}$$

with the following addition and scalar multiplication.

$$(1, y_1) + (1, y_2) = (1, y_1^2 + y_2^2)$$

and

$$k(1, y) = (1, ky)$$

If it is not a vector space, clearly state and show which axiom fails.

Question 4. Let $\mathscr{W} = \{p(x) = a_0 + a_2x^2 + a_3x^3 \mid p(-1) = 0\}$ be a vector subspace of P_3.

a. *(4 marks)* Find a basis S for $\mathscr{W}$.

b. *(2 marks)* Determine the dimension of $\mathscr{W}$, Justify.

c. *(2 marks)* Find the coordinate vector of $p(x) = -2 + 2x^2$ relative to the basis S.

Question 5. (4 marks) Find all values of z such that the triangle with vertices $A(-1,2,-3)$, $B(4,-5,6)$ and $C(1,-2,z)$ has area equal to 31.

Question 6. *(3 marks)* Prove: If V be a vectorspace, $\vec{0}$ the zero vector of V, k a scalar then $k\vec{0} = \vec{0}$. **Show EVERY step and justify EVERY step with axiom names when an axiom is used, you may use the fact that $0\vec{u} = \vec{0}$.**

Question 7. *(4 marks)* Given

$$\vec{u}_1 = (1,2,3), \quad \vec{u}_2 = (3,2,k), \quad \vec{v}_1 = (1,-2,3), \quad \vec{v}_2 = (2,3,t)$$

find all values of $t,\ k$ such that $\operatorname{span}(\{\vec{u}_1,\ \vec{u}_2\}) \subseteq \operatorname{span}(\{\vec{v}_1,\ \vec{v}_2\})$

Bonus Question. *(5 marks)* In a vector space, the span of any subset is a subspace. In addition, the span of a subset of a vector space is the smallest subspace containing all vectors of the subset.